Homework Solutions

§6.2

#2. (a) $\det\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = 14-15 = -1 \neq 0$ so $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ is nonsingular.

(b) $\det\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} = -12-(-12) = 0$ so $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$ is nonsingular

(c) $\det\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \\ 0 & 4 & -3 \end{bmatrix} = 1\det\begin{bmatrix} 0 & 1 \\ 4 & -3 \end{bmatrix} - 2\det\begin{bmatrix} 2 & -1 \\ 4 & -3 \end{bmatrix}$

$= 1(0-4) - 2(-6+4) = -4+4 = 0$

so $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \\ 0 & 4 & -3 \end{bmatrix}$ is singular.

#4. (a) $A = \begin{bmatrix} 0 & -1 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$ $\det A = 0 + 1\det\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} + 2\det\begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$

$= 0 + 1(1-1) + 2(-1-0) = -2 \neq 0$

the columns of A are linearly independent.

(b) $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ $\det A = 1\det\begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix} - 0 + 2\det\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$

$= 1(-3-1) + 2(2) = -4+4 = 0$

the columns of A are linearly dependent.

#5. $\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = q.$

(a) $\begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix} \xrightarrow{r_2 \leftrightarrow r_3} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ so $\det\begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix} = -1(q)$

$= -q.$

(b) $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \xrightarrow{r_1 \to 3r_1} \begin{bmatrix} 3a & 3b & 3c \\ d & e & f \\ g & h & i \end{bmatrix}$. This matrix has determinant $3q$.

$\det = q$ $\qquad\qquad$ $\det = 3q$

(c) $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \xrightarrow{r_3 - 4r_1 \to r_3} \begin{bmatrix} a & b & c \\ d & e & f \\ g-4a & h-4b & i-4c \end{bmatrix}$ This matrix has determinant equal to q.

(d) $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_2} \begin{bmatrix} d & e & f \\ a & b & c \\ g & h & i \end{bmatrix}$ $\times -1$

$\xrightarrow{r_2 \leftrightarrow r_3} \begin{bmatrix} d & e & f \\ g & h & i \\ a & b & c \end{bmatrix}$ $\times -1$

$\xrightarrow{2r_3 \to 2r_3} \begin{bmatrix} d & e & f \\ g & h & i \\ 2a & 2b & 2c \end{bmatrix}$ $\times 2$

$\xrightarrow{r_3 + 3r_2 \to r_2} \begin{bmatrix} d & e & f \\ g & h & i \\ 2a+3g & 2b+3h & 2c+3i \end{bmatrix}$ This matrix has determinant $(-1)(-1)(2)q$

$= 2q$.

#7. Given A & B are $n \times n$ & AB is nonsingular. ($\det(AB) \neq 0$)

Show: A & B are both nonsingular, i.e. $\det A \neq 0$ & $\det B \neq 0$.

Proof: Because AB is nonsingular we know $\det(AB) \neq 0$.

But $\det(AB) = \det(A)\det(B) \neq 0$ means that neither $\det A$ nor $\det B$ can be zero. Hence both A & B are nonsingular. $\boxed{\checkmark}$