Homework Solutions

§6.1

#2. (a) $\det \begin{bmatrix} 2 & -5 \\ 3 & 4 \end{bmatrix} = 2(4) - (-5)(3) = 8 + 15 = 23$

(b) $\det \begin{bmatrix} 1 & 3 & 2 \\ -2 & 3 & 5 \\ 1 & 2 & 4 \end{bmatrix} = 1 \det \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} - 3 \det \begin{bmatrix} -2 & 5 \\ 1 & 4 \end{bmatrix} + 2 \det \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$

$= 1(12-10) - 3(-8-5) + 2(-4-3)$

$= 2 + 39 - 14$

$= 27$

(c) $\det \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 5 \\ 1 & 3 & 5 \end{bmatrix} = 3 \det \begin{bmatrix} 2 & 5 \\ 3 & 5 \end{bmatrix} - 1 \det \begin{bmatrix} 1 & 5 \\ 1 & 5 \end{bmatrix} + 0 \cdots$

$= 3(10-15) - 1(5-5) + 0$

$= -15 - 0 + 0$

$= -15$

(d) $\det \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = 1 \det \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - 2 \det \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + 3 \det \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$= 1 \cdot 0 - 2 \cdot 0 + 3 \cdot 0$

$= 0.$

#4. $\det M = \det \begin{bmatrix} 2 & k & 1 \\ 2 & 1 & k \\ 3 & k & -1 \end{bmatrix} = 2 \det \begin{bmatrix} 1 & k \\ k & -1 \end{bmatrix} - k \det \begin{bmatrix} 2 & k \\ 3 & -1 \end{bmatrix} + 1 \det \begin{bmatrix} 2 & 1 \\ 3 & k \end{bmatrix}$

$= 2(-1-k^2) - k(-2-3k) + 1(2k-3)$

$= -2 - 2k^2 + 2k + 3k^2 + 2k - 3$

$= k^2 + 4k - 5$

$= (k+5)(k-1)$

So $\det M = 0$ when $k = -5$ or $k = 1$.

#5. (a) $\det \begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 4 \end{bmatrix} = -0 + 3\det \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix} - 0 = 3(4-2) = 6.$

(b) $\det \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 3 & 5 \end{bmatrix} = 0 - 0 + 5\det \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = 5(2-1) = 5.$

(c) $\det \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 2 & 0 \\ 1 & 2 & 0 & 3 \\ 0 & 0 & 2 & -1 \end{bmatrix} = -0 + 0 - 2\det \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 0 & 0 & -1 \end{bmatrix}$

$= -2\left(0 - 0 + (-1)\det \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}\right) = -2(0 - 0 - (0)) = 0.$

#7. $M = \begin{bmatrix} 2-k & 2 & 0 \\ 0 & 3-k & 2 \\ 0 & 0 & 1+k \end{bmatrix}$ * M is upper triangular.

$\det(M) = (2-k)(3-k)(1+k) = 0$ when $k = 2$, $k = 3$, or $k = -1$.