Homework Solutions

§5.4

#3. (a) $\{v_1, v_2\}$ or $\{v_1, v_3\}$ or $\{v_2, v_3\}$.
because each pair is independent and spans $\mathbb{R}^2$.

(b) $\{v_1, v_3\}$ or $\{v_2, v_3\}$
because v_1 & v_2 are parallel

(c) These 3 vectors are linearly independent in $\mathbb{R}^3$, so they span $\mathbb{R}^3$. $\{v_1, v_2, v_3\}$ is a basis.

(d) $-2v_1 + 3v_2 = v_3$ so ~~either~~ both $\{v_1, v_3\}$ or $\{v_2, v_3\}$ ~~is a~~ are bases.

#4. (a) $\text{nul}(A) = \{0\}$, the origin; $\text{col}(A) = \mathbb{R}^2$, the x-y plane.

(b) $\text{nul}(B) = \text{Span}\left\{\begin{bmatrix}1\\-1\end{bmatrix}\right\}$, the line $y=-x$; $\text{col}(B) = \text{Span}\left\{\begin{bmatrix}1\\1\end{bmatrix}\right\}$, the line $y=x$

(c) $\text{nul}(C) = \text{Span}\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\right\}$, the x-axis; $\text{col}(C) = \text{Span}\left\{\begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$, the y-z plane

(d) $\text{nul}(D) = \{\text{vectors on line through origin, normal to plane}\} = \text{Span}\left\{\begin{bmatrix}2\\-3\\1\end{bmatrix}\right\}$
$\text{col}(D) = \{\text{the plane}\}$.

(e) $\text{nul}(E) = \{\text{xy-plane}\} = \text{Span}\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$
$\text{col}(E) = \{z\text{-axis}\} = \text{Span}\left\{\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$.

#7. (a) (i) basis for col(A) is $\{v_1, v_3\}$. ← columns of $\underline{A}$

$v_2 = -2v_1$, $v_4 = 2v_1$, $v_5 = 2v_3 - v_1$

(ii) $\{v_1 v_3\}$, $\{v_3, v_4\}$, $\{\ldots$

(b) (i) basis for col(A) is $\{v_1, v_2, v_4\}$.

$v_3 = -v_1 - 2v_2$

(ii) $\{v_1, v_3, v_4\}$, $\{v_2, v_3, v_4\}$.

#8. When A is $n \times n$ & invertible:

1. $Ax = 0 \iff x = 0$ so $\text{nul}(A) = \{0\}$
2. $Ax = b$ is consistent for all $b \in \mathbb{R}^n$ so $\text{col}(A) = \mathbb{R}^n$ (or columns of A span $\mathbb{R}^n$)
3. A invertible $\iff A^T$ invertible; by 2. $\text{col}(A^T) = \mathbb{R}^n$. Then $\text{row}(A) = \text{col}(A^T) = \mathbb{R}^n$.

#9. (a) False. A 4x3 matrix can have no free variables

(b) True. A 3x4 matrix must have at least one free variable.

(c) True. A 4x4 w/ $\text{nul}(A) = 0$ means A have 4 pivots, i.e. $\text{col}(A) = \mathbb{R}^4$.

(d) False. A 4x3 w/ $\text{nul}(A) = 0$ means no free variables; 3 pivots. The 3 columns cannot be a basis for a 4 dim. subspace.

(e) False. You can have 3 lin. indep. vectors in $\mathbb{R}^4$.

(f) True. The 4 rows in $\mathbb{R}^3$ must be lin. dependent.

(g) False. The row & col. space have the same dimension, but not the same spaces.

(h) False. Only the null space & row space ~~ag~~ are the same.