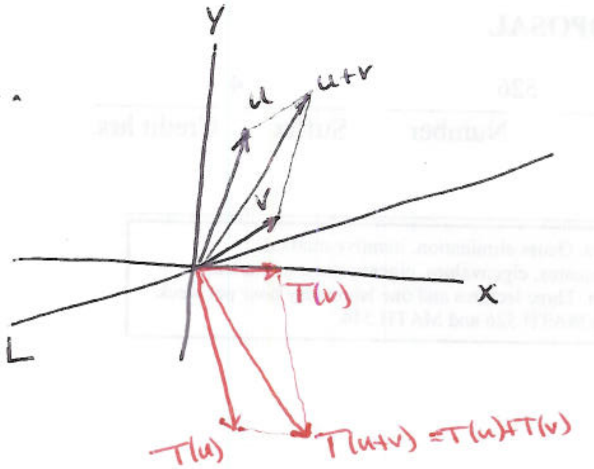
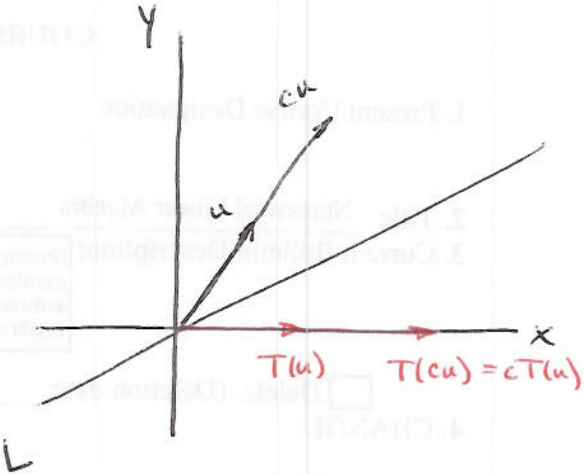
Homework Solutions

§4.4

#1.





Reflecting u+v gives the same vector as reflecting u & v separately and then adding.
Likewise, reflecting cu gives the same vector as reflecting u and then multiplying by c.

#2. $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}2\\-1\\3\end{bmatrix}$ and $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}4\\5\\0\end{bmatrix}$

so $T(x) = Ax$ with $A = \begin{bmatrix}2 & 4\\-1 & 5\\3 & 0\end{bmatrix}$.

#3. To show au+bv is in the kernel of the linear transformation T:

$$T(au+bv) = T(au)+T(bv) = aT(u)+bT(v) = 0$$

Note: You do not need the specific values of a, b, u & v — these make the problem harder!

#5. We have to prove 2 statements:

1. if T is one-to-one then kernel of T is $\{0\}$.

Proof: Suppose x is in the kernel of T. Then $T(x)=0$.

But, we also know that $T(0)=0$.

Because T is one-to-one, the output 0 can come from only one input vector. Thus $x=0$. $\boxtimes$

2. if the kernel of T is $\{0\}$ then T is one-to-one.

Proof: Suppose T is not one-to-one.

Then there must be 2 vectors x & y with $T(x)=T(y)$.

But, then $T(x-y)=T(x)-T(y)=0$

and so $x-y$ must be in the kernel of T.

Since the only vector in the kernel of T is 0:

$x-y=0$

and so $x=y$

This contradicts the assumption that T is not one-to-one $\boxtimes$

#7. We know that both T & S are linear transformations. This means $T(x+y)=T(x)+T(y)$, $T(cx)=cT(x)$, $S(x+y)=S(x)+S(y)$, and $S(cx)=cS(x)$.

Now:

$$(T\circ S)(x+y)=T(S(x+y))=T(S(x)+S(y))=T(S(x))+T(S(y))$$
$$=(T\circ S)(x)+(T\circ S)(y).$$

and

$$(T\circ S)(cx)=T(S(cX))=T(cS(x))=cT(S(x))=c(T\circ S)(x).$$

Therefore $T\circ S:\mathbb{R}^n\to\mathbb{R}^k$ is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^k$.