Homework Solutions

§4.3

#1. (a) Using the standard unit vector method to find M:

- M leaves the y-axis fixed, so $Me_2 = e_2$.
- M rotates the z-axis to the x-axis: $Me_3 = e_1$
- M rotates the x-axis to the neg. z-axis: $Me_1 = -e_3$.

Thus: $M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$.

(b) Using the standard unit vector method to find M:

- M leaves the x-axis fixed: $Me_1 = e_1$
- M maps the $+$ z-axis to the $-$ z-axis: $Me_3 = -e_3$
- M maps the $+$ y-axis to the $-$ y-axis: $Me_2 = -e_2$.

Thus: $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(c) Using the standard unit vector method to find M:

- M leaves the x- & z- axes unchanged: $Me_1 = e_1$, $Me_3 = e_3$.
- M maps the $+$ y-axis to the $-$ y-axis: $Me_2 = -e_2$.

Thus: $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

#3. (a) $A = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 3 & -1 \end{bmatrix}$ Note that A is in echelon form.

To solve $A\underset{\sim}{x} = 0$ we have
$$2x_1 + x_2 - 3x_3 = 0$$
$$3x_2 - x_3 = 0.$$

Let x_3 be the free variable.

#3(a) cont.

Then $x_2 = \frac{1}{3}x_3$

and $x_1 = \frac{1}{2}(3x_3 - x_2) = \frac{1}{2}\left(\frac{8}{3}x_3\right) = \frac{4}{3}x_3$.

and the general solution to $Ax=0$ is

$$x = x_3 \begin{bmatrix} 4/3 \\ 1/3 \\ 1 \end{bmatrix}.$$

The kernel of $T(x) = Ax$ is $\text{Span}\left\{\begin{bmatrix} 4/3 \\ 1/3 \\ 1 \end{bmatrix}\right\} = \text{Span}\left\{\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}\right\}$.

(b). If $T(x) = Bx : \mathbb{R}^3 \to \mathbb{R}^2$ (i.e. B is a 2×3 matrix).

then there must be at least one free variable when we reduce the augmented matrix $[B \mid 0]$ to echelon form.

Because there is a free variable, there are non-trivial solutions to $Bx=0$ & so the kernel of T contains a nonzero vector.

Note: We could also state that ~~there~~ the columns of B are linearly dependent (because any set of k vectors in $\mathbb{R}^n$ with $k>n$ must be linearly dependent) and so there is a linear dependence relation between the columns, i.e. there is a nonzero solution to $Bx=0$.