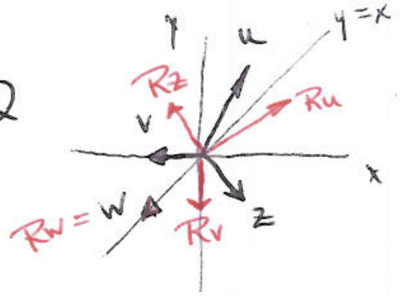
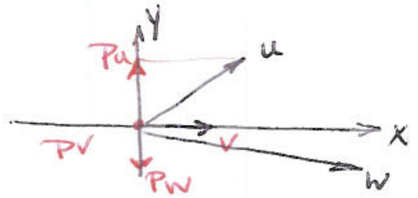
Homework Solutions
§4.1

#2

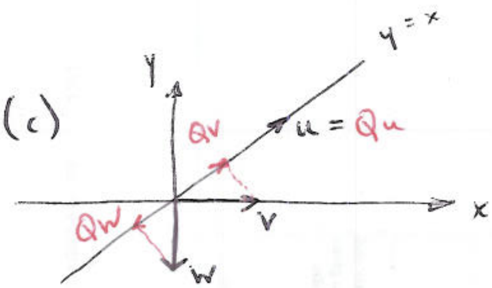


#4. (a)



(b) Every vector on the x-axis gets mapped to the 0 vector. The kernel is the entire x-axis. All outputs will be on the y-axis; this is the range.

(c)

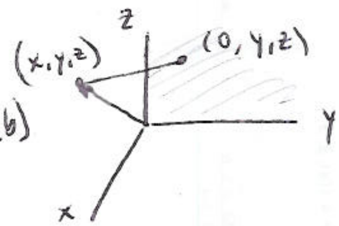


(d) Every vector on the line $y=-x$ is mapped to the 0 vector. The kernel is the line $y=-x$.

All outputs will be on the line $y=x$. The range of Q is the line $y=x$.

#5(a) $Ax = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ b \\ c \end{bmatrix}$ This is a general vector in the y-z plane

(b)



(c) Kernel of T $= \left\{ \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix} : x \in \mathbb{R} \right\}$ = x-axis

Range of T $= \left\{ \begin{bmatrix} 0 \\ y \\ z \end{bmatrix} : y, z \in \mathbb{R} \right\}$ = y, z plane

#6. (a) $A \in \mathbb{R}^{m \times n}$, $0 \in \mathbb{R}^n$ $T(0) = A0 = 0 \in \mathbb{R}^m$ so $0 \in \mathbb{R}^n$ is in kernel of T.

(b) $u, v \in$ kernel of T means $Au=0$ & $Av=0$. Now, $\cancel{A}(u+v) = Au + Av = 0 + 0 = 0$
so $u+v$ is also in the kernel of T.

(c). $u \in$ kernel of T & $c \in \mathbb{R}$ so $T(cu) = A(cu) = c(Au) = c0 = 0$.
This means cu is also in the kernel of T.