Homework Solutions
§3.6

#1. (a) Claim: When $A^3 = 0$, $(I-A)^{-1} = I + A + A^2$.

Proof: $(I-A)(I+A+A^2) = I + A + A^2 - A - A^2 - A^3$
$= I - A^3$
$= I$

$(I+A+A^2)(I-A) = I - A + A - A^2 + A^2 - A^3$
$= I - A^3$
$= I$

This means $I - A$ is invertible and $(I-A)^{-1} = I + A + A^2$. ∎

#2. (a) B is obtained from A with the row operation: $(r_2 - 3r_1 \to r_2)$.

Thus $E = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

(b) B is obtained from A by interchanging rows 1 & 2.
Thus $E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) B is obtained from A by the row operation: $r_3 + 2r_1 \to r_3$

Thus $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$.

#3. $A = \begin{bmatrix} -1 & 3 \\ 4 & -6 \end{bmatrix} \xrightarrow{r_2 + 4r_1 \to r_2} \begin{bmatrix} -1 & 3 \\ 0 & 6 \end{bmatrix}$ $\quad E_1 = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$

$\xrightarrow{\frac{1}{6} r_2} \begin{bmatrix} -1 & 3 \\ 0 & 1 \end{bmatrix} \xrightarrow{r_1 - 3r_2 \to r_1} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{-r_1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1/6 \end{bmatrix}$

So $E_4 E_3 E_2 E_1 A = I$. This means $A^{-1} = E_4 E_3 E_2 E_1$ $\quad E_3 = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1/6 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{2}{3} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} 1 & 1/2 \\ 2/3 & 1/6 \end{bmatrix}$ $\quad E_4 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$