Homework Solutions

§3.5

#1. (a) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}^{-1} = \dfrac{1}{2\cdot 3 - 5\cdot 1}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}.$

#3 (a)

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ -2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{r_2+2r_1 \to r_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 2 & 3 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{array}\right] \xrightarrow{r_2 \leftrightarrow r_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 2 \end{array}\right]$$

$$\xrightarrow{r_3-2r_2 \to r_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{array}\right] \xrightarrow[r_1-r_3 \to r_1]{r_2-r_3 \to r_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{array}\right]$$

$$\xrightarrow{r_1-r_2 \to r_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{array}\right] \quad \text{So} \quad \begin{bmatrix} 0 & 1 & 1 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ 3 & -1 & -2 \\ -2 & 1 & 2 \end{bmatrix}$$

Check: $\begin{bmatrix} 0 & 1 & 1 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 1 \\ 3 & -1 & -2 \\ -2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ✓.

#4 (a) $3AX = 2B - 4C$

$AX = \frac{2}{3}B - \frac{4}{3}C$

$X = A^{-1}\left(\frac{2}{3}B - \frac{4}{3}C\right)$ assuming A is invertible.

(b) $3AX = 2B - 4X$

$3AX + 4X = 2B$

$(3A + 4I)X = 2B$

$X = (3A+4I)^{-1}(2B)$ assuming $3A+4I$ is invertible.

#8. (a) if A is invertible, then it follows that <u>the columns of A</u> are linearly independent

(b) if A is a square matrix and the equation $Ax=0$ <u>has nontrivial solutions</u> then the matrix A is singular.

(c) if A is square and <u>its columns are linearly independent</u> then A is nonsingular.

(d) True.

(e) The third column of $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 4 & 6 \\ 3 & 4 & 7 \end{bmatrix}$ is <u>a linear combination of columns 1 & 2</u> and therefore A does not have an inverse

#10. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $B = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$AB = \frac{1}{ad-bc}\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} ad-bc & -ab+ba \\ cd-cd & -bc+ad \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} ad-bc & bd-bd \\ -ca+ca & -bc+ad \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

so $B = A^{-1}$.

#13. Because A is invertible we can write $B = (A^{-1}A)B = A^{-1}(AB)$. Now the product of invertible matrices is invertible so that $A^{-1}(AB)$ is invertible and its inverse must be B^{-1}. Thus:

$$B^{-1} = (A^{-1}(AB))^{-1} = (AB)^{-1}(A^{-1})^{-1} = (AB)^{-1}A. \quad \square$$