

Homework Solutions

§3.3

$$\#1. 3A - AB + 12C - 4CB = A(3I - B) + 4C(3I - B) \\ = (A + 4C)(3I - B).$$

$$\#2. Cx + 5d = 4x \iff Cx - 4x = -5d \iff (C - 4I)x = -5d. \\ \text{so } A = C - 4I \text{ and } b = -5d.$$

$$\#3(a) (A - B)(A + B) = A(A + B) - B(A + B) \\ = AA + AB - BA - BB \\ = A^2 + AB - BA - B^2. = A^2 - B^2 \text{ only when } AB = BA.$$

Counter-example: $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}.$

$$A - B = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \quad A + B = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix} \quad (A - B)(A + B) = \begin{bmatrix} 2 & 2 \\ -4 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad B^2 = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \quad A^2 - B^2 = \begin{bmatrix} 0 & -2 \\ -4 & 0 \end{bmatrix}.$$

$$\#5. 4A^T + 2B = 3C^T$$

$$4A^T = 3C^T - 2B$$

$$A^T = \frac{3}{4}C^T - \frac{1}{2}B$$

$$(A^T)^T = \left(\frac{3}{4}C^T - \frac{1}{2}B \right)^T$$

$$A = \frac{3}{4}C - \frac{1}{2}B^T.$$