Homework Solutions

§3.1

#2. $4\begin{bmatrix}2\\1\\7\end{bmatrix}+5\begin{bmatrix}0\\1\\2\end{bmatrix}=\begin{bmatrix}8\\9\\38\end{bmatrix}$ can be written as $\begin{bmatrix}2&0\\1&1\\7&2\end{bmatrix}\begin{bmatrix}4\\5\end{bmatrix}=\begin{bmatrix}8\\9\\38\end{bmatrix}$.

#3. $v=\begin{bmatrix}1\\2\\-1\end{bmatrix}$ so that Av has column 1 + 2 × column 2 − column 3.

#5. $A\in\mathbb{R}^{2\times3}$ with rows v_1 & v_2, nonzero & nonparallel.
if $Ax=0$ then $v_1\cdot x=0$ & $v_2\cdot x=0$.
Using the interpretation of dot product, this means x is perpendicular to both v_1 & v_2. That is x is ~~in the spanned~~ normal to the plane spanned by v_1 & v_2.

Also, if v is in the plane spanned by v_1 & v_2, then $v=av_1+bv_2$.
Then $x\cdot v=a(x\cdot v_1)+b(x\cdot v_2)=a(0)+b(0)=0$ which shows that x is normal to the plane spanned by v_1 & v_2.

#7(b) N is 30×3, each row is of the form $[\,t_1\ \ t_2\ \ e\,]$.
To weight the two midterm tests 25% each and the final exam 50%, use $w=\begin{bmatrix}.25\\.25\\.50\end{bmatrix}$.

#9. (a) True, $b=Ap$ so b can be written as -2 col. 1 + col. 2 + 4 col 4.
(b) Cannot determine. Need to know if $Ax=0$ has any nontrivial solutions.
(c) Cannot determine. (same answer as (b)).
(d) True. $Ap=b$ means that $2b=2Ap=A(2p)$, so $x=2p=\begin{bmatrix}-4\\2\\0\\8\end{bmatrix}$.
(e) False. From (a): $-2A_1+A_2+4A_4=b$ so $-2A_1+A_2+4A_4-b=0$ is a linear dependence relation for A_1, A_2, A_3, A_4 & b.

#10 (a) If the matrix vector equation has
only the trivial solution,
then the columns of A are linearly independent.

(b) If the matrix-vector equation $Ax=b$ is consistent
for all vectors b in $\mathbb{R}^5$, then
the columns of A
span all of $\mathbb{R}^5$.

(c) If the solution set of the matrix-vector equation
$Ax=0$ has a free variable, then the
columns of A form a linearly dependent set.

(d) If the matrix-vector equation $Ax=0$ has
nontrivial solutions,
then the columns of A are linearly dependent.