Homework Solutions

§2.5

#2.

$$2\begin{bmatrix}-2\\1\\-1\\-1\end{bmatrix}+3\begin{bmatrix}2\\1\\1\\1\end{bmatrix}+\begin{bmatrix}3\\-1\\3\\2\end{bmatrix}+3\begin{bmatrix}0\\0\\1\\-1\end{bmatrix}=\begin{bmatrix}5\\4\\7\\0\end{bmatrix}.$$

x_1 coeff of x_1 $\quad$ x_2 coeff of x_2 $\quad$ x_3 coeff of x_3 $\quad$ x_4 coeff of x_4 $\quad$ right-hand side

#4.

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}1\\-1\\-1\\0\end{bmatrix}+x_4\begin{bmatrix}-5\\-7\\4\\1\end{bmatrix}$$

To not use the 1st column, $x_1=0$: $1-5x_4=0 \Rightarrow x_4=1/5$.

With $x_4=1/5$: $x_1=0$, $x_2=-12/5$, $x_3=-1/5$, $x_4=1/5$.

So,

$$0\begin{bmatrix}3\\1\\2\end{bmatrix}-\frac{12}{5}\begin{bmatrix}1\\1\\1\end{bmatrix}-\frac{1}{5}\begin{bmatrix}5\\0\\0\end{bmatrix}+\frac{1}{5}\begin{bmatrix}2\\2\\3\end{bmatrix}=\begin{bmatrix}-3\\-2\\3\end{bmatrix}$$

To use all 4 columns there are many possible choices for x_4: $\left(\begin{array}{l}x_4\neq 1/5\\x_4\neq -1/7\\x_4\neq 1/4\\x_4\neq 0\end{array}\right)$

With $x_4=1$: $x_1=-4$, $x_2=-8$, $x_3=3$, $x_4=1$.

So

$$-4\begin{bmatrix}3\\1\\2\end{bmatrix}-8\begin{bmatrix}1\\1\\1\end{bmatrix}+3\begin{bmatrix}5\\0\\0\end{bmatrix}+\begin{bmatrix}2\\2\\3\end{bmatrix}=\begin{bmatrix}-3\\-2\\3\end{bmatrix}$$

#5 We must solve $c_1v_1+c_2v_2+c_3v_3=0$.

$$\begin{aligned}-c_1-3c_2+4c_3&=0\\2c_1+c_2-c_3&=0\\3c_1-4c_2-c_3&=0\\c_1+4c_2+2c_3&=0\end{aligned}\qquad\begin{bmatrix}-1&-3&4&0\\2&1&-1&0\\3&-4&-1&0\\1&4&2&0\end{bmatrix}$$

#6(c)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ 0 \\ 3 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$$= \text{Span}\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

#7. Let w be a linear combination of $v_1, \dots, v_k$.

This means that it is possible to write

$$w = c_1 v_1 + c_2 v_2 + \dots + c_k v_k$$

for some choice of $c_1, c_2, \dots, c_k$.

Then $c_1 v_1 + c_2 v_2 + \dots + c_k v_k - w = 0$.

But, this is just a linear dependence relation for $\{v_1, v_2, \dots, v_k, w\}$.

(note that at least one coefficient is not zero — which one?)

Therefore, because a linear dependence relation exists for $\{v_1, \dots, v_k, w\}$,

the vectors $\{v_1, \dots, v_k, w\}$ is linearly dependent.