Homework Solutions

§2.1

#5. Line 1: $x = t$, $y = 4$ $\quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad p = \begin{bmatrix} 0 \\ 4 \end{bmatrix}, t = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Line 2: $x = 2$, $y = t$ $\quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\quad p = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, t = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Line 3: $y = 2x$ $\quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 2x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + x \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\quad p = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, t = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

#8. $v = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$, $u = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$. Direction of line is $v - u = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$.

The vector form of the line is $\begin{bmatrix} 1 \\ -3 \end{bmatrix} + t \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

(You can use any other point on the line, e.g. $\begin{bmatrix} 6 \\ 4 \end{bmatrix} + t \begin{bmatrix} 5 \\ 7 \end{bmatrix}$.)

#9. $x = 2t + 1$, $y = t + 3$, $z = t$ $\quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ $\quad p = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$.

#11. $A = (2, 0, 4)$, $B = (1, 3, -2)$, $C = (-1, 9, -15)$

$u = \overrightarrow{AB} = \begin{bmatrix} -1 \\ 3 \\ -6 \end{bmatrix}$

$v = \overrightarrow{AC} = \begin{bmatrix} -3 \\ 9 \\ -19 \end{bmatrix}$

$w = \overrightarrow{BC} = \begin{bmatrix} -2 \\ 6 \\ -13 \end{bmatrix}$

If all three points lie on a single line, then they must all be able to be written in the form $p + tv$ with $p = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$, $v = \begin{bmatrix} -1 \\ 3 \\ -6 \end{bmatrix}$.

- with $t = 0$: $p + tv = p + v = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$
- to have $p + tv = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ means $tv = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -6 \end{bmatrix}$ so $t = 1$.
- to have $p + tv = \begin{bmatrix} -1 \\ 9 \\ -15 \end{bmatrix}$ mean $tv = \begin{bmatrix} -1 \\ 9 \\ -15 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} -3 \\ 9 \\ -19 \end{bmatrix}$

but this has no solutions as we cannot have $-t = -3$, $3t = 9$, and $-6t = -19$.