```
> restart;
> with( VLA ):
```

▼ Instructions for Use

- You will need some of these results on the paper-and-pencil exam that will be given Wednesday, February 7, 2007.
- For each matrix without parameters,
  (i) find the echelon form of the matrix and
  (ii) find all solutions (not none, if the linear system is inconsistent.
- For each matrix with parameters,
  (i) find the echelon form of the matrix and
  (ii) be prepared to determine when the system is consistent or inconsistent, ...
- Note that you have to determine which matrix is relevant for each problem. Some may not be used for any problem.
- Note that every vector and matrix has a unique name, you can use these to help refer to results in this worksheet when you are working the rest of the exam on Wednesday.
- Save your work, and print the worksheet BEFORE coming to class for the exam on Wednesday.

```
>
```

▼ Matrices without parameters

```
> M1 := Matrix( [[1,2,1,0],[2,4,3,1]] );
```

$$M1 := \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 3 & 1 \end{bmatrix} \tag{2.1}$$

```
> Reduce( M1 );
```

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{2.2}$$

```
> Backsolve( (2.2) );
```

$$\begin{bmatrix} -1-2X_2 \\ X_2 \\ 1 \end{bmatrix} \tag{2.3}$$

```
>
> M2 := Matrix( [[1,2], [2,4], [1,3], [0,1]] );
```

$$M2 := \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 1 & 3 \\ 0 & 1 \end{bmatrix} \tag{2.4}$$

```
> Reduce( M2 );
```

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{2.5}$$

```
> Backsolve( (2.5) );
```

No solutions (2.6)

```
>
> M3 := Matrix( [[1,3,1,2], [2,6,3,4], [3,9,1,6] ] );
```

$$M3 := \begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 3 & 4 \\ 3 & 9 & 1 & 6 \end{bmatrix} \tag{2.7}$$

```
> Reduce( M3 );
```

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.8}$$

```
> Backsolve( (2.8) );
```

$$\begin{bmatrix} 2-3X_2 \\ X_2 \\ 0 \end{bmatrix} \tag{2.9}$$

```
>
> M4 := Matrix( [[1,2,3], [3,6,9], [1,3,1], [2,4,6] ] );
```

$$M4 := \begin{bmatrix} 1 & 2 & 3 \\ 3 & 6 & 9 \\ 1 & 3 & 1 \\ 2 & 4 & 6 \end{bmatrix} \tag{2.10}$$

```
> Reduce( M4 );
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.11}$$

```
> Backsolve( (2.11) );
```

$$\begin{bmatrix} 7 \\ -2 \end{bmatrix} \tag{2.12}$$

```
>
> M5 := Matrix( [[1,3,1,2], [2,6,2,5], [3,9,3,6] ] );
```

$$M5 := \begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 2 & 5 \\ 3 & 9 & 3 & 6 \end{bmatrix} \tag{2.13}$$

```
> Reduce( M5 );
```

(2.14)

$$\begin{vmatrix} 1 & 3 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{vmatrix} \quad (2.14)$$

```
> Backsolve( (2.14) );
```

No solutions (2.15)

```
>
> M6 := Matrix( [[1,1,1,3], [9,3,1,6], [4,1,0,-1]] );
```

$$M6 := \begin{vmatrix} 1 & 1 & 1 & 3 \\ 9 & 3 & 1 & 6 \\ 4 & 1 & 0 & -1 \end{vmatrix} \quad (2.16)$$

```
> Reduce( M6 );
```

$$\begin{vmatrix} 1 & 1 & 1 & 3 \\ 0 & -6 & -8 & -21 \\ 0 & 0 & 0 & -\frac{5}{2} \end{vmatrix} \quad (2.17)$$

```
> Backsolve( (2.17) );
```

No solutions (2.18)

```
>
> M7 := Matrix( [[1,1,1,3], [9,3,1,6]] );
```

$$M7 := \begin{vmatrix} 1 & 1 & 1 & 3 \\ 9 & 3 & 1 & 6 \end{vmatrix} \quad (2.19)$$

```
> Reduce( M7 );
```

$$\begin{vmatrix} 1 & 1 & 1 & 3 \\ 0 & -6 & -8 & -21 \end{vmatrix} \quad (2.20)$$

```
> Backsolve( (2.20) );
```

$$\begin{vmatrix} -\frac{1}{2} + \frac{1}{3} X_3 \\ \frac{7}{2} - \frac{4}{3} X_3 \\ X_3 \end{vmatrix} \quad (2.21)$$

```
>
> M8 := Matrix( [[1,1,1,3], [9,3,1,6], [4,1,0,3/2]] );
```

$$M8 := \begin{vmatrix} 1 & 1 & 1 & 3 \\ 9 & 3 & 1 & 6 \\ 4 & 1 & 0 & \frac{3}{2} \end{vmatrix} \quad (2.22)$$

```
> Reduce( M8 );
```

$$\begin{vmatrix} 1 & 1 & 1 & 3 \\ 0 & -6 & -8 & -21 \\ 0 & 0 & 0 & 0 \end{vmatrix} \quad (2.23)$$

```
> Backsolve( (2.23) );
```

$$\begin{vmatrix} -\frac{1}{2} + \frac{1}{3} X_3 \\ \frac{7}{2} - \frac{4}{3} X_3 \\ X_3 \end{vmatrix} \quad (2.24)$$

```
>
> M9 := Matrix( [[-1,-3,4,0], [2,1,-1,0], [3,-4,-1,0], [1,-4,2,0]] );
```

$$M9 := \begin{vmatrix} -1 & -3 & 4 & 0 \\ 2 & 1 & -1 & 0 \\ 3 & -4 & -1 & 0 \\ 1 & -4 & 2 & 0 \end{vmatrix} \quad (2.25)$$

```
> Reduce( M9 );
```

$$\begin{vmatrix} -1 & -3 & 4 & 0 \\ 0 & -5 & 7 & 0 \\ 0 & 0 & -\frac{36}{5} & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \quad (2.26)$$

```
> Backsolve(  (2.26));
```

$$\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} \quad (2.27)$$

```
>
> M10 := Matrix( [[0,3,3,6,0], [3,1,4,2,0], [-1,1,0,2,0], [0,2,2,4,0], [1,
  -1,0,-2,0]] );
```

$$M10 := \begin{vmatrix} 0 & 3 & 3 & 6 & 0 \\ 3 & 1 & 4 & 2 & 0 \\ -1 & 1 & 0 & 2 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 1 & -1 & 0 & -2 & 0 \end{vmatrix} \quad (2.28)$$

```
> Reduce( M10 );
```

(2.29)

$$\begin{bmatrix} 3 & 1 & 4 & 2 & 0 \\ 0 & 3 & 3 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.29}$$

```
> Backsolve( (2.29) );
```

$$\begin{bmatrix} -X_3 \\ -X_3 - 2X_4 \\ X_3 \\ X_4 \end{bmatrix} \tag{2.30}$$

```
>
>
```

▼ Matrices with parameters

```
> N1 := Matrix( [ [1,1,1,1], [1,2,2,k], [1,3,h,5]]);
```

$$N1 := \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & k \\ 1 & 3 & h & 5 \end{bmatrix} \tag{3.1}$$

```
> Reduce( N1 );
```

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & k-1 \\ 0 & 0 & h-3 & 6-2k \end{bmatrix} \tag{3.2}$$

```
>
>
> N2 := Matrix( [ [1,1,1,1], [1,2,h,k], [1,3,2,5]]);
```

$$N2 := \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & h & k \\ 1 & 3 & 2 & 5 \end{bmatrix} \tag{3.3}$$

```
> Reduce( N2 );
```

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & h-1 & k-1 \\ 0 & 0 & 3-2h & 6-2k \end{bmatrix} \tag{3.4}$$

```
>
>
> N3 := Matrix( [ [1,1,1,1], [1,2,h,2], [1,3,2,k]]);
```

$$N3 := \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & h & 2 \\ 1 & 3 & 2 & k \end{bmatrix} \tag{3.5}$$

```
> Reduce( N3 );
```

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & h-1 & 1 \\ 0 & 0 & 3-2h & k-3 \end{bmatrix} \tag{3.6}$$

```
>
>
> N4 := Matrix( [ [1,1,1,1], [1,2,2,5], [1,3,h,k]]);
```

$$N4 := \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 5 \\ 1 & 3 & h & k \end{bmatrix} \tag{3.7}$$

```
> Reduce( N4 );
```

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & h-3 & k-9 \end{bmatrix} \tag{3.8}$$

```
>
>
>
```