Extra Credit Lab

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Overview

Here are two extra problems. Each problem is worth ten (10) homework points, if completed successfully. All solutions must be turned in no later than the *beginning* of the final exam.

Question 1 – Blue Whale Model

Studies of the blue whale population conducted in the 1930's led to the following model.

Becasue female blue whales can produce a calf only once during a two-year period, the age classes are assumed to be: less than 2 years, 2 or 3 years, 4 or 5 years, 6 or 7 years, 8 or 9 years, 10 or 11 years, and 12 or more years. The size of each subpopulation from one year to the next can be modeled by $\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k$ where the matrix for the model is

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0.19 & 0.44 & 0.50 & 0.50 & 0.45 \\ 0.77 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.77 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.77 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.77 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.77 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.77 & 0.78 \end{bmatrix}$$

1. Explain the meaning of each non-zero entry in $\mathbf{A}$.
2. Does this model predict the blue whale population will become extinct?

NOTE: *Use topics discussed in this course (viz., Lab 13) to explain your answer.*

Question 2 – Hill Substitution Cipher

Download and read the information about Hill Substitution Ciphers at the URL[1]
`http://www.math.sc.edu/~meade/math526/misc/hillcipher.pdf`.

1. Use the key matrix
$$\mathbf{A} = \begin{bmatrix} 3 & 10 & 20 \\ 20 & 9 & 17 \\ 9 & 4 & 17 \end{bmatrix}$$
to decode "gjsdepokohxtnuasum" and "gjsdepokopgtnxkimzizilfdokowwtwfdfkrhbwyvjmyhqaucxx".
NOTE: *Blanks have been removed from the strings and z's have been added to the end of the string to ensure the final length is a multiple of 3.*
2. Answer the questions contained in these phrases. NOTE: *Use Roman numerals to represent numbers.*
3. Find a new key matrix and use it to encode the responses found in 2. Explain why you know your matrix is a key matrix. (Do not use any of the matrices in the attached document. Make your own key matrix. Students who use the same key matrix will not receive credit for this problem.)
4. Submit your encoded responses <u>and</u> the key matrix to me.

You do <u>not</u> need to compute the inverse of any matrix modulo 26. The reference document describes the process. `MATLAB` does not have a command to find the inverse of a matrix modulo 26; Maple[2] does:

```
> with( LinearAlgebra:-Modular ):
> A := < < 3, 20, 9> | <10, 9, 4> | <20, 17, 17> >;   # key matrix (def'd by columns)
> Ainv := Inverse(26,A);                              # inverse, mod 26
```

[1] This document is part of *Linear Algebra and Its Applications*, Third Edition, by David C. Lay. The original URL for this document is `http://media.pearsoncmg.com/aw/aw_lay_linearalg_updated_cw_3/cs_apps/hillcipher.pdf`.
[2] Maple is available in the computers labs in LeConte, PSC, Swearingen, and many other computer labs across campus.