The Power Method for Finding Eigenvalues

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Overview

Eigenvalues have many different uses in mathematics, engineering, and the sciences. One application of eigenvalues will be explored in next week's lab. This week's lab focuses on the computation of eigenvalues via the Power Method.

The new `MATLAB` command introduced in this lab is `eig`, which computes the eigenvalues and eigenvectors for a square matrix.

Part I

Let $\mathbf{A}$ be a square matrix. A number λ is an eigenvalue of $\mathbf{A}$ if and only if there is a *nonzero* vector $\mathbf{v}$ such that $\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$. The vector $\mathbf{v}$ is called an eigenvector of $\mathbf{A}$ corresponding to the eigenvalue λ.

The `eig` Command

Create the matrix $\mathbf{A} = \begin{bmatrix} -1.5 & -1.5 & -1.5 \\ 2 & 2 & 1 \\ -1 & -1 & -0.5 \end{bmatrix}$.

Then, enter and execute the following `MATLAB` commands:

≫ eig(A)	**% eigenvalues of A**
≫ [V,D] = eig(A)	**% eigenvalues and eigenvectors of A**

The information in the eigenvalue decomposition of a matrix can be extracted and verified as follows:

≫ lambda1 = D(1,1)	**% an eigenvalue of A**
≫ v1 = V(:,1)	**% the corresponding eigenvector of A**
≫ A*v1, lambda1*v1	**% verify result**

Verify that the remaining two eigenvalue-eigenvector pairs of $\mathbf{A}$ are correct.

Repeated Eigenvalues

Every $n \times n$ matrix has n eigenvalues. Some eigenvalues can be complex-valued but if the matrix is real-valued, the complex eigenvalues will appear in complex conjugate pairs. That is, if $\lambda = a + ib$ is an eigenvalue then $\bar{\lambda} = a - ib$ is another eigenvalue of the matrix. Most matrices have n distinct eigenvalues. The infrequent matrices with repeated eigenvalues need to be noted and worked with cautiously.

Use `MATLAB` to find all eigenvalues, and the corresponding eigenvectors, for the following matrices:

$$\mathbf{A} = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 5 & -6 \\ 7.5 & 11 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 5 & -3 & 0 & 9 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 5 & -3 & 0 & 9 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

Repeat the following commands for several matrices of different sizes. It is a rare event to find an $n \times n$ matrix that does not have n eigenvalues.

≫ M = rand(3)	**% random 3×3 matrix**
≫ eig(M)	**% how many eigenvalues?**

Clear all variables before you begin to work on Part II.