# HW #1 – Solutions

#0.1 Equation (0.15) is the equation that gives the percentage of users on page 2 at the next snapshot.

- A user currently on page 1 can get to page 2 by typing the page 2 URL or by clicking on the page 2 URL (1 of 2 URLs on page 1). This contributes:

$$0.15(0.25)\,x_1^{old} + 0.85(0.5)\,x_1^{old} = \frac{37}{80}\,x_1^{old}$$

- A user currently on page 2 remains on page 2 only if the page 2 URL is typed:

$$\frac{3}{20}\cdot\frac{1}{4}\,x_2^{old} = \frac{3}{80}\,x_2^{old}$$

- A user currently on page 3 ends up on page 2 by typing the page 2 URL or by clicking on the page 2 URL (1 of 3 URLs):

$$\frac{3}{20}\cdot\frac{1}{4}\,x_3^{old} + \frac{17}{20}\cdot\frac{1}{3}\,x_3^{old} = \frac{77}{240}\,x_3^{old}$$

- A user currently on page 4 goes to page 2 only if the page 2 URL is typed:

$$\frac{3}{20}\,\frac{1}{4}\,x_4^{old} = \frac{3}{80}\,x_4^{old}.$$

Thus $x_2^{new} = \frac{37}{80}\,x_1^{old} + \frac{3}{80}\,x_2^{old} + \frac{77}{240}\,x_3^{old} + \frac{3}{80}\,x_4^{old}$ (this is (0.15)).

Equations (0.16) and (0.17) are derived in a similar manner. Note that the only way to click to page 3 is from page 1 (1 of 2 URLs). A user can click to page 4 from either page 2 (1 of 2 URLs) or page 3 (1 of 3 URLs).

#0.2. All components in the matrix A are positive because there is a way to get from one page to any other page - by typing the URL. (An entry would be zero if there were no way to get from a page to another page.)

Also, no entry is negative because these entries are all probabilities and probabilities are always found in [0,1].

#0.5 The sum of the entries (components) in the steady-state distribution is the sum of the probabilities that the user is on a page. Since each user must be on one of these pages, the total probability is 1. (One interpretation is that no browsers crashed or hung - the group of users is fixed.)

#1.1. $\vec{v}_1 = \begin{bmatrix} 4 \\ 0 \\ -2 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ -2 \\ 8 \\ -1 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 6 \end{bmatrix}$

$$-\vec{v}_1 + 3\vec{v}_2 - 2\vec{v}_3 = \begin{bmatrix} -4+6+6 \\ 0-6-2 \\ 2+24 \\ -1-3-12 \end{bmatrix} = \begin{bmatrix} 8 \\ -8 \\ 26 \\ -16 \end{bmatrix}$$

#1.2. $A = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 3 & -2 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}$

$C = \begin{bmatrix} -5 & 1 & 2 \\ 4 & -5 & -3 \end{bmatrix}$ $D = \begin{bmatrix} 0 & -2 & 5 \\ -2 & 1 & -3 \\ 4 & 2 & -1 \end{bmatrix}$ $w = \begin{bmatrix} -3 & 5 \end{bmatrix}$.

(a) $$A\vec{v} = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -1 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} +3+2-12 \\ -12-1+4 \end{bmatrix} = \begin{bmatrix} -7 \\ -9 \end{bmatrix}$$

$$wB = \begin{bmatrix} -3 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 0 \\ -2 & 3 & -2 \end{bmatrix} = \begin{bmatrix} -9-10 & 3+15 & 0-10 \end{bmatrix}$$

$$= \begin{bmatrix} -19 & 18 & -10 \end{bmatrix}.$$

$$Dv = \begin{bmatrix} 0 & -2 & 5 \\ -2 & 1 & -3 \\ 4 & 2 & -1 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0-2+20 \\ 6+1-12 \\ -12+2-4 \end{bmatrix} = \begin{bmatrix} 18 \\ -5 \\ -14 \end{bmatrix}$$

$$v^tD = \begin{bmatrix} -3 & 1 & 4 \end{bmatrix} \begin{bmatrix} 0 & -2 & 5 \\ -2 & 1 & -3 \\ 4 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 0-2+16 & 6+1+8 & -15-3-4 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 15 & -22 \end{bmatrix}$$

$$Aw^t = \begin{bmatrix} -1 & 2 & -3 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 5 \end{bmatrix}$$ is not defined; you cannot multiply a 2x3 matrix and a 2x1 vector.

$$DC^t = \begin{bmatrix} 0 & -2 & 5 \\ -2 & 1 & -3 \\ 4 & 2 & -1 \end{bmatrix} \begin{bmatrix} -5 & 4 \\ 1 & -5 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 0-2+10 & 0+10-15 \\ 10+1-6 & -8-5+9 \\ -20+2-2 & 16-10+3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -5 \\ 5 & -4 \\ -20 & 9 \end{bmatrix}$$

(b) $$2A-3B = \begin{bmatrix} -2 & 4 & -6 \\ 8 & -2 & 2 \end{bmatrix} + \begin{bmatrix} -9 & 3 & 0 \\ 6 & -9 & 6 \end{bmatrix} = \begin{bmatrix} -11 & 7 & -6 \\ 14 & -11 & 8 \end{bmatrix}$$

(c) $$AD = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -2 & 5 \\ -2 & 1 & -3 \\ 4 & 2 & -1 \end{bmatrix} = \begin{bmatrix} -16 & -2 & -8 \\ 6 & -7 & 22 \end{bmatrix}$$

DB is not defined because D has 3 columns but B has only 2 rows (need to have # ~~rows~~ columns in D = # rows in B)

(d) $$A+B = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 0 \\ -2 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -3 \\ 2 & 2 & -1 \end{bmatrix}$$

$$B+A = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 3 & -2 \end{bmatrix} + \begin{bmatrix} -1 & 2 & -3 \\ 4 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -3 \\ 2 & 2 & -1 \end{bmatrix}$$

same result.

(e) $$(A+B)+C = \begin{bmatrix} 2 & 1 & -3 \\ 2 & 2 & -1 \end{bmatrix} + \begin{bmatrix} -5 & 1 & 2 \\ 4 & -5 & -3 \end{bmatrix} = \begin{bmatrix} -3 & 2 & -1 \\ 6 & -3 & -4 \end{bmatrix}$$

$$A+(B+C) = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 0 & 2 \\ 2 & -2 & -5 \end{bmatrix} = \begin{bmatrix} -3 & 2 & -1 \\ 6 & -3 & -4 \end{bmatrix}$$

same result.

(f) $2(A+B) = 2\begin{bmatrix} 2 & 1 & -3 \\ 2 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 2 & -6 \\ 4 & 4 & -2 \end{bmatrix}$ ← same result

$2A+2B = \begin{bmatrix} -2 & 4 & -6 \\ 8 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 0 \\ -4 & 6 & -4 \end{bmatrix} = \begin{bmatrix} 4 & 2 & -6 \\ 4 & 4 & -2 \end{bmatrix}$

(g) $(3B)^t = \begin{bmatrix} 9 & -3 & 0 \\ -6 & 9 & -6 \end{bmatrix}^t = \begin{bmatrix} 9 & -6 \\ -3 & 9 \\ 6 & -6 \end{bmatrix}$

$3B^t = 3\begin{bmatrix} 3 & -2 \\ -1 & 3 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 9 & -6 \\ -3 & 9 \\ 0 & -6 \end{bmatrix}$ ← Same result

(h) $(A+B)^t = \begin{bmatrix} 2 & 1 & -3 \\ 2 & 2 & -1 \end{bmatrix}^t = \begin{bmatrix} 2 & 2 \\ 1 & 2 \\ -3 & -1 \end{bmatrix}$ ← same result

$A^t + B^t = \begin{bmatrix} -1 & 4 \\ 2 & -1 \\ -3 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 3 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 1 & 2 \\ -3 & -1 \end{bmatrix}$

#1.3 $A_{m \times n}\, v_{n \times 1}$ will be an $m \times 1$ matrix (i.e. a column-vector).

#1.4. $v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $w = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$v^t w = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 4+10+18 \end{bmatrix} = \begin{bmatrix} 32 \end{bmatrix}$

$vw^t = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \quad \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}.$