> restart;

Maple Worksheet to Accompany Text on p. 93 of Peressini, Sullivan, Uhl

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WWW URL: http://www.math.sc.edu/~meade/math524/worksheets/newton.ms

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In this example we want to use Newton's method to find the minimizer of the function

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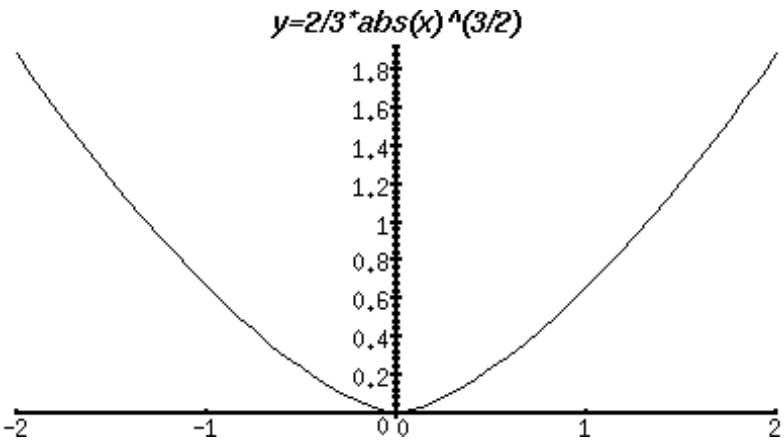
> f := x-> 2/3*abs(x)^(3/2);

$$f := x \to \frac{2}{3}\left| x \right|^{3/2}$$

> plot(f, -2..2, title='y=2/3*abs(x)^(3/2)');



Note that the minimizer is also a zero of the function. It is therefore possible to compare the Newton's method for a zero of a function and for a minimizer. Denote the iterates looking for a zero by { x[k] } and those for the minimizer by { xx[k] }. Thus:

x[k+1] = x[k] - f(x[k]) / f'(x[k])

and

xx[k+1] = xx[k] - f'(xx[k]) / f''(xx[k])

Do you understand the distinction between the two sequences of iterates?

>

The first and second derivatives of f are needed before defining the iterates. (Since f is defined as a function, we use the D operator to compute the derivatives of f.)

```
> df := D(f):
  df2 := D(df):
> newtonZ := x -> x-f(x)/df(x);
```

$$newtonZ := x \to x - \frac{f(x)}{df(x)}$$

```
> newtonM := x -> x-df(x)/df2(x);
```

$$newtonM := x \to x - \frac{df(x)}{df2(x)}$$

```
>
```

And, now, the iterates for the zero of f --- note that these converge (rapidly) to 0.

```
> x[0] := 1;
  for k from 0 to 10 do
    x[k+1] := newtonZ( x[k] )
  od;
```

$$x_0 := 1$$

$$x_1 := \frac{1}{3}$$

$$x_2 := \frac{1}{9}$$

$$x_3 := \frac{1}{27}$$

$$x_4 := \frac{1}{81}$$

$$x_5 := \frac{1}{243}$$

$$x_6 := \frac{1}{729}$$

$$x_7 := \frac{1}{2187}$$

$$x_8 := \frac{1}{6561}$$

$$x_9 := \frac{1}{19683}$$

$$x_{10} := \frac{1}{59049}$$

$$x_{11} := \frac{1}{177147}$$

And now the iterates for the minimizer of f -- these exhibit the behavior described in the text.

```
> xx[0] := 1;
  for k from 0 to 10 do
    xx[k+1] := newtonM( xx[k] )
  od;
```

$$xx_0 := 1$$

$$xx_1 := -1$$

$$xx_2 := 1$$

$$xx_3 := -1$$

$$xx_4 := 1$$

$$xx_5 := -1$$

$$xx_6 := 1$$

$$xx_7 := -1$$

$$xx_8 := 1$$

$$xx_9 := -1$$

$$xx_{10} := 1$$

$$xx_{11} := -1$$

>

Do you expect to see the same type of behavior for different starting values of the sequences? (TRY IT!)

Why do the second set of iterates behave so poorly? (HINT: plot the functions df and df2)

END OF WORKSHEET

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