MATH 524 University of South Carolina
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Calculus and Linear Algebra Review Problems

1. Let $f(x, y, z) = xy^3 - 5x^2yz^4$. Compute ∇f.

2. In what direction is $f(x, y) = 9x^4 + 4y^2$ increasing most rapidly at $(1, 2)$?

3. Define $f(t) = \|\mathbf{F}(t)\|$ where $\mathbf{F}$ is a differentiable function. Compute $f'(t)$.

4. Find and classify all critical points of $f(x, y) = xy^2 - 6x^2 - 3y^2$ and of $g(x, y) = e^{-(x^2+y^2-4y)}$.

5. A rectangular box whose edges are parallel to the coordinate axes is inscribed in the ellipsoid $36x^2 + 4y^2 + 9z^2 = 36$. What is the greatest possible volume for the box?

6. Are the vectors
$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \qquad \mathbf{v}_3 = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}.$$
linearly dependent or independent? Are these vectors a basis for $\mathbf{R}^3$?

7. Find the inverse of the matrix
$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

8. Let A and B be invertible $n \times n$ matrices. What is the inverse of their product, AB?

9. Define $A^\dagger = (A^T A)^{-1} A^T$. Compute $A^\dagger$, $A^\dagger A$, and $AA^\dagger$ when
$$A = \begin{pmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$