Take Home Exam

Solve the following 4 problems. It is not necessary to evaluate all integrals for Fourier coefficients – but you should see the values of coefficients that can be determined by observation.

1. $\nabla^2 u = 0 \qquad 1 < r < 2, \quad 0 < \theta < 2\pi$

$$\left.\begin{array}{l} u(1,\theta) = \sin(3\theta) \\ \frac{\partial u}{\partial r}(2,\theta) = \cos(2\theta) \end{array}\right\} \quad 0 < \theta < 2\pi$$

2. $\nabla^2 u = 0 \qquad 0 < x < \pi, \quad 0 < y < \pi$

$u(0,y) = 0, \ u(\pi,y) = 1 \qquad 0 < y < \pi$

$u_y(x,0) = \cos(3x), \ u_y(x,\pi) = 0 \qquad 0 < x < \pi$

3. $u_{tt} = 4u_{xx} \qquad 0 < x < 3, \quad t > 0$

$u(x,0) = \sin(\pi x) \qquad 0 < x < 3$

$u_t(x,0) = 0$

$u(0,t) = u(3,t) = t^3 \qquad t > 0$

Hint: Find the solution in Regions I & II; for extra credit, find the solution in regions IV.

4. $u_{tt} = u_{xx} + u_{yy} \qquad 0 < x < 1, \ 0 < y < \pi, \ t > 0$

$u(x,0,t) = u(x,\pi,t) = 0 \qquad 0 < x < 1, \ t > 0$

$u_x(0,y,t) = u_x(1,y,t) = 0 \qquad 0 < y < \pi, \ t > 0$

$$\left.\begin{array}{l} u(x,y,0) = y\cos\left(\frac{\pi x}{2}\right) \\ u_t(x,y,0) = x + y \end{array}\right\} \quad 0 < x < 1, \ 0 < y < \pi$$