HW Solution - §7.1

#14. $x_1' = a_{11}x_1 + a_{12}x_2 + g_1(t)$ $\quad x_1(0) = x_1^0$
$x_2' = a_{21}x_1 + a_{22}x_2 + g_2(t)$ $\quad x_2(0) = x_2^0$.

If $a_{12} \neq 0$: $x_2 = \frac{1}{a_{12}}\left(x_1' - a_{11}x_1 - g_1(t)\right)$

so $x_2' = \frac{1}{a_{12}}\left(x_1'' - a_{11}x_1' + g_1'(t)\right)$.

Plugging these expressions for x_2 & x_2' into the 2nd DE gives:

$$a_{12} * \left[\frac{1}{a_{12}}\left(x_1'' - a_{11}x_1' + g_1'(t)\right) = a_{21}x_1 + \frac{a_{22}}{a_{12}}\left(x_1' - a_{11}x_1 - g_1(t)\right) + g_2(t)\right]$$

$$\boxed{x_1'' - (a_{11}+a_{22})x_1' + (a_{11}a_{22} - a_{12}a_{21})\,x_1 = -g_1'(t) - a_{22}\,g_1(t) + a_{12}\,g_2(t)}$$

The corresponding IC are:

$\boxed{x_1(0) = x_1^0}$

and $\cancel{x_2^0 = x_2(0) = \frac{1}{a_{12}}\left(x_1'(0) - a_{11}x_1(0) - g_1(0)\right) = \frac{1}{a_{12}}\left(x_1'(0) - a_{11}x_1^0 - g_1(0)\right)}$

← correct, but unnecessary, just plug $t=0$ into the 1st DE!

~~so~~ $\boxed{x_1'(0) = a_{11}x_1^0 + a_{12}x_2^0 + g_1(0)}$

If $a_{12} = 0$, then $a_{21} \neq 0$ & do the corresponding simplifications for x_2.

Note that if the coefficients are functions of t, with $a_{12}(t)$ or $a_{21}(t)$ never zero then the same steps are still valid - except that the computation of x_2' will involve the quotient rule, which will be very tedious and messy - but very doable with enough patience.