HW Solutions - §4.1

#17. Show that $W[5, \sin^2 t, \cos 2t] = 0$ for all t.

Direct evaluation:

$$W[5, \sin^2 t, \cos 2t] = \det \begin{bmatrix} 5 & \sin^2 t & \cos 2t \\ 0 & 2\sin t\cos t & -2\sin 2t \\ 0 & 2\cos^2 t - 2\sin^2 t & -4\cos 2t \end{bmatrix}$$

$$\left\{ \begin{array}{l} \sin 2t = 2\sin t\cos t \\ \cos 2t = \cos^2 t - \sin^2 t \end{array} \right\}$$

$$= 5 \det \begin{bmatrix} 2\sin t\cos t & -2\sin 2t \\ 2(\cos^2 t - \sin^2 t) & -4\cos 2t \end{bmatrix}$$

$$= 5\left((2\sin t\cos t)(-4\cos 2t) + 2(\cos^2 t - \sin^2 t) \cdot (+2)(\sin 2t) \right)$$

$$= 5\left(-4\sin 2t\cos 2t + 4\cos 2t\sin 2t\right)$$

$$= 0.$$

Without direct evaluation:

$$\begin{aligned} \cos 2t &= \cos^2 t - \sin^2 t \\ &= (1 - \sin^2 t) - \sin^2 t \\ &= 1 - 2\sin^2 t \\ &= \tfrac{1}{5}(5) - 2(\sin^2 t) \end{aligned}$$

so one of the functions is a linear combination of the other two.

This means the 3 functions are linearly dependent, and so the Wronskian must be zero.