Homework Solutions - §3.3

#23. $3u'' - u' + 2u = 0 \qquad u(0) = 2,\ u'(0) = 0$

(a) $3r^2 - r + 2 = 0 \qquad r = \dfrac{1 \pm \sqrt{1 - 4\cdot 3\cdot 2}}{2\cdot 3} = \dfrac{1 \pm \sqrt{23}\, i}{6}$

$y_1 = e^{t/6} \cos \frac{\sqrt{23}}{6} t \qquad y_2 = e^{t/6} \sin \frac{\sqrt{23}}{6} t$

$\underline{y = c_1 e^{t/6} \cos \frac{\sqrt{23}}{6} t + c_2 e^{t/6} \sin \frac{\sqrt{23}}{6} t}$

To satisfy the IC: $\qquad y(0) = \underline{c_1 = 2}$

$y' = c_1 \left(\frac{1}{6} e^{t/6} \cos \frac{\sqrt{23}}{6} t - \frac{\sqrt{23}}{6} e^{t/6} \sin \frac{\sqrt{23}}{6} t \right)$
$\quad + c_2 \left(\frac{1}{6} e^{t/6} \sin \frac{\sqrt{23}}{6} t + \frac{\sqrt{23}}{6} e^{t/6} \cos \frac{\sqrt{23}}{6} t \right)$

$y'(0) = \frac{c_1}{6} + \frac{\sqrt{23}}{6} c_2 = 0$

$\Rightarrow \underline{c_2 = \frac{6}{\sqrt{23}} \left(-\frac{2}{6} \right) = -\frac{2}{\sqrt{23}}}.$

$\therefore \underline{y = 2 e^{t/6} \cos \frac{\sqrt{23}}{6} t - \frac{2}{\sqrt{23}} e^{t/6} \sin \frac{\sqrt{23}}{6} t}$

(b) There is no analytic way to solve the equation $|u(t)| = 10$. What is reasonable is to graph $u(t)$ and look for the first time $u(t) = 10$ or $u(t) = -10$. Doing this you find that $u(\underline{10.7598}) \approx -10$.

#31. Let $r = \lambda + i\mu$.

$\frac{d}{dt} e^{rt} = \frac{d}{dt} \left(e^{\lambda t} \cos \mu t + i e^{\lambda t} \sin \mu t \right)$

$= \lambda e^{\lambda t} \cos \mu t - \mu e^{\lambda t} \sin \mu t + i \left(\lambda e^{\lambda t} \sin \mu t + \mu e^{\lambda t} \cos \mu t \right)$

$= (\lambda + i\mu) e^{\lambda t} \cos \mu t + (-\mu + i\lambda) e^{\lambda t} \sin \mu t$

$= (\lambda + i\mu) e^{\lambda t} \cos \mu t + i(\lambda + i\mu) e^{\lambda t} \sin \mu t$

$= (\lambda + i\mu) \left(e^{\lambda t} \cos \mu t + i e^{\lambda t} \sin \mu t \right)$

$= r e^{rt}.$