§ 2.6 Homework Solutions

#5. $\frac{dy}{dx} = -\frac{ax+by}{bx+cy}$ can be rewritten as $\underbrace{(ax+by)}_{M} + \underbrace{(bx+cy)}_{N.}\frac{dy}{dx} = 0$.

To check if this DE is exact:

$$\left.\begin{array}{l}\frac{\partial M}{\partial y} = b \\ \frac{\partial N}{\partial x} = b\end{array}\right\}$$

∴ exact DE; we look for the sol'n in the implicit form $\psi(x,y) = C$

where $\psi_x = M = ax+by \Rightarrow \psi = \frac{a}{2}x^2 + bxy + g(y)$

& $\psi_y = N = bx+cy$ $\quad \psi_y = bx + g'(y)$

$cy = g'(y)$ ∴ $g = \frac{c}{2}y^2$

The implicit sol'n is: $\boxed{\frac{a}{2}x^2 + bxy + \frac{c}{2}y^2 = K.}$

#6. $\frac{dy}{dx} = -\frac{ax-by}{bx-cy}$ is easily rewritten as $\underbrace{ax-by}_{M} + \underbrace{(bx-cy)}_{N.}\frac{dy}{dx} = 0$

Check if exact: $\cancel{\frac{\partial M}{\partial x} = a \text{ and } \frac{\partial N}{\partial y} = -b}$

$\frac{\partial M}{\partial y} = -b$ but $\frac{\partial N}{\partial x} = +b$

∴ this DE is not exact!

#20. $\left(\frac{\sin y}{y} - 2e^{-x}\sin x\right) + \frac{\cos y + 2e^{-x}\cos x}{y}\, y' = 0$

To show this is not exact, look at $\frac{\partial M}{\partial y} = \frac{y\cos y - 1\cdot\sin y}{y^2} = \frac{y\cos y - \sin y}{y^2}$.

and $\frac{\partial N}{\partial x} = \frac{2}{y}e^{-x}(-\sin x) - \frac{2}{y}e^{-x}\cos x$

Since M_y & N_x are not equal, this DE is not exact.

Given an integrating factor $\mu = ye^x$, which allows us to rewrite the DE as: $\underbrace{(e^x\sin y - 2y\sin x)}_{M} + \underbrace{(e^x\cos y + 2\cos x)}_{N}\frac{dy}{dx} = 0.$

Thus: $\frac{\partial M}{\partial y} = e^x\cos y - 2\sin x$ & $\frac{\partial N}{\partial x} = e^x\cos y - 2\sin x$ ∴ exact.

Sol'n is $\psi(x,y) = C$ where

$\psi_x = e^x\sin y - 2y\sin x \Rightarrow \psi = e^x\sin y + 2y\cos x + h(y)$

& $\psi_y = e^x\cos y - 2\cos x$ $\quad \psi_y = e^x\cos y + 2\cos x + h'(y)$

$h'(y) = 0$ so $h(y) = C$ & sol'ns are $\boxed{e^x\sin y + 2y\cos x = C.}$