MATH 520 (Section 001) Prof. Meade

Exam 2 March 8, 2013 University of South Carolina Spring 2013

## Instructions:

- 1. There are a total of 5 problems (including the Extra Credit problem) on 2 pages. Check that your copy of the exam has all of the problems.
- 2. You may bring one notecard (not a full sheet of paper) on which you have written (i) the factorization of one polynomial and (ii) the solution of one linear system.
- 3. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and should not involve unreasonable manual calculations.
- 4. Be sure you answer the questions that are asked.
- 5. You must show all of your work to receive full credit for a correct answer. Correct answers with no supporting work will be eligible for at most half-credit.
- 6. Your answers must be clearly labeled and written legibly on additional sheets of paper (that I will provide). Be sure each sheet contains your name and the work for each question is clearly labeled.
- 7. Check your work. If I see *clear evidence* that you checked your answer (when possible) <u>and</u> you *clearly indicate* that your answer is incorrect, you will be eligible for more points than if you had not checked your work.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Extra Credit	10	
Total	100	

Enjoy Spring Break!

- 1. (20 points) Consider the differential equation  $t^2y''' + 8ty'' + (\tan t)y = \cos(t)$ . Do not attempt to solve this equation.
  - (a) What is the order of this differential equation? Is this differential equation linear or nonlinear? Does this differential equation have constant coefficients? Could you find a fundamental set of solutions to the corresponding homogeneous differential equation? Explain.
  - (b) Determine the longest interval in which the solution to the initial value problem for this differential equation with the initial conditions y(5) = 2, y'(5) = -1, y''(5) = 0 is certain to have a unique solution.
  - (c) What does Abel's Theorem tell us about the Wronskian of a fundamental set of solutions to this differential equation?
- 2. (20 points) Find the solution of the initial value problem

$$y'' + 2y' + 5y = 0$$
,  $y(0) = 2$ ,  $y'(0) = 4$ .

- 3. (20 points) Consider the differential equation  $y'' 6y' + 9y = \frac{e^{3t}}{t}$ .
  - (a) Find the general solution to the corresponding homogeneous equation.
  - (b) Use the method of variation of parameters to find a particular solution.
- 4. (20 points) Find the solution of the initial value problem

$$2y^{(4)} - y''' - 9y'' + 4y' + 4y = 0, \qquad y(0) = y''(0) = 2, \quad y'(0) = y'''(0) = 0.$$

5. (20 points) Consider the differential equation

$$y^{(4)} + 4y'' = 3\sin(t) + 10te^t + 8.$$

Determine a suitable form for a particular solution found by the method of undetermined coefficients. *Do not solve for the coefficients.* 

Extra Credit (10 points) Determine a particular solution to the nonhomogeneous equation in #5.