

>

Maple Worksheet for Qualitative Analysis for Nonlinear Systems (Section 6.7 in Ledder)

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>

```
> restart;
> with( DEtools );
> with( plots );
> with( LinearAlgebra );
> with( plottools );
>
> SYS2 := [
>   diff( x(t), t ) = f( x(t), y(t) ),
>   diff( y(t), t ) = g( x(t), y(t) )
> ];
>
> SYS3 := [
>   diff( x(t), t ) = f( x(t), y(t), z(t) ),
>   diff( y(t), t ) = g( x(t), y(t), z(t) ),
>   diff( z(t), t ) = h( x(t), y(t), z(t) )
> ];
>
>
>
```

▶ # 2 (p. 408)

▶ # 3 (p. 408)

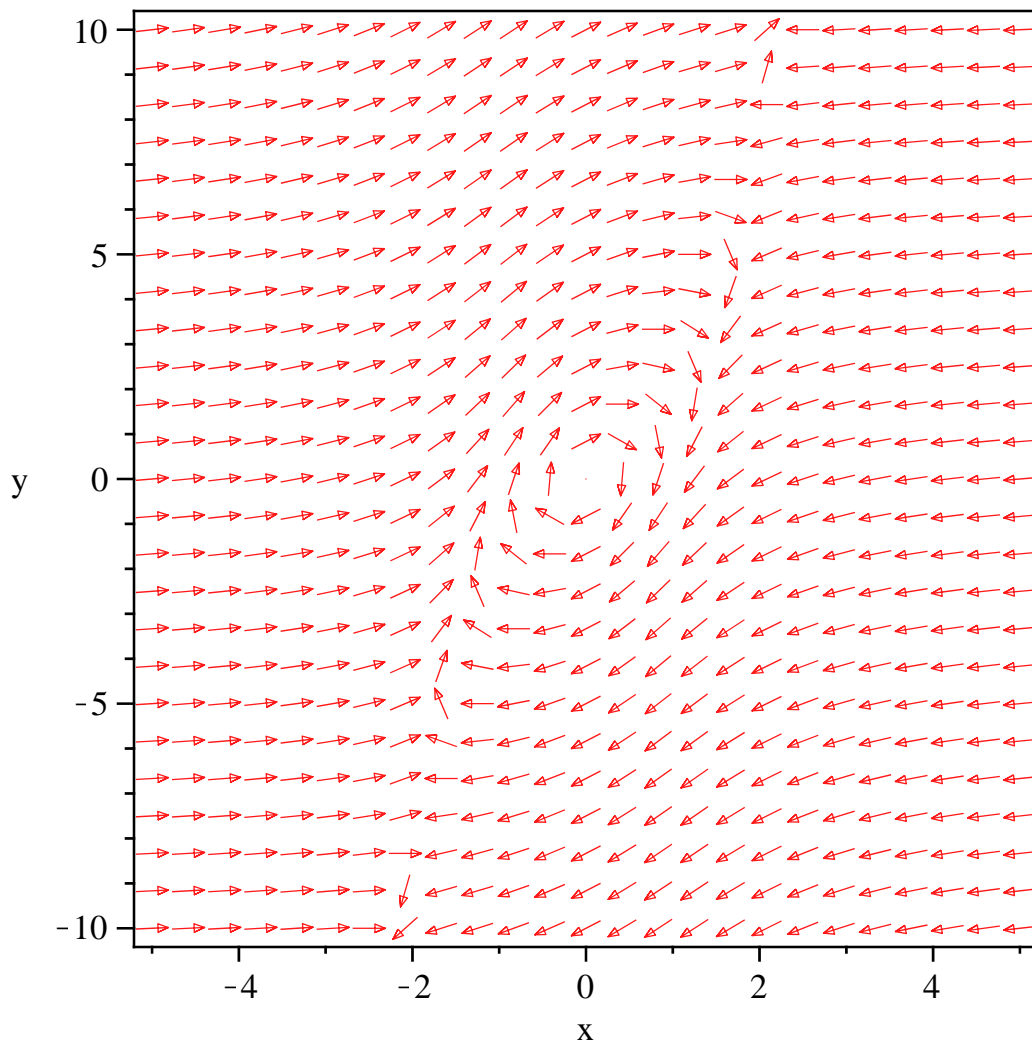
▼ # 6 (p. 408)

```
> f := (x,y) -> y-x^3;
> g := (x,y) -> y-4*x;
```

$$\begin{aligned} f &:= (x, y) \rightarrow y - x^3 \\ g &:= (x, y) \rightarrow y - 4x \end{aligned}$$

(3.1)

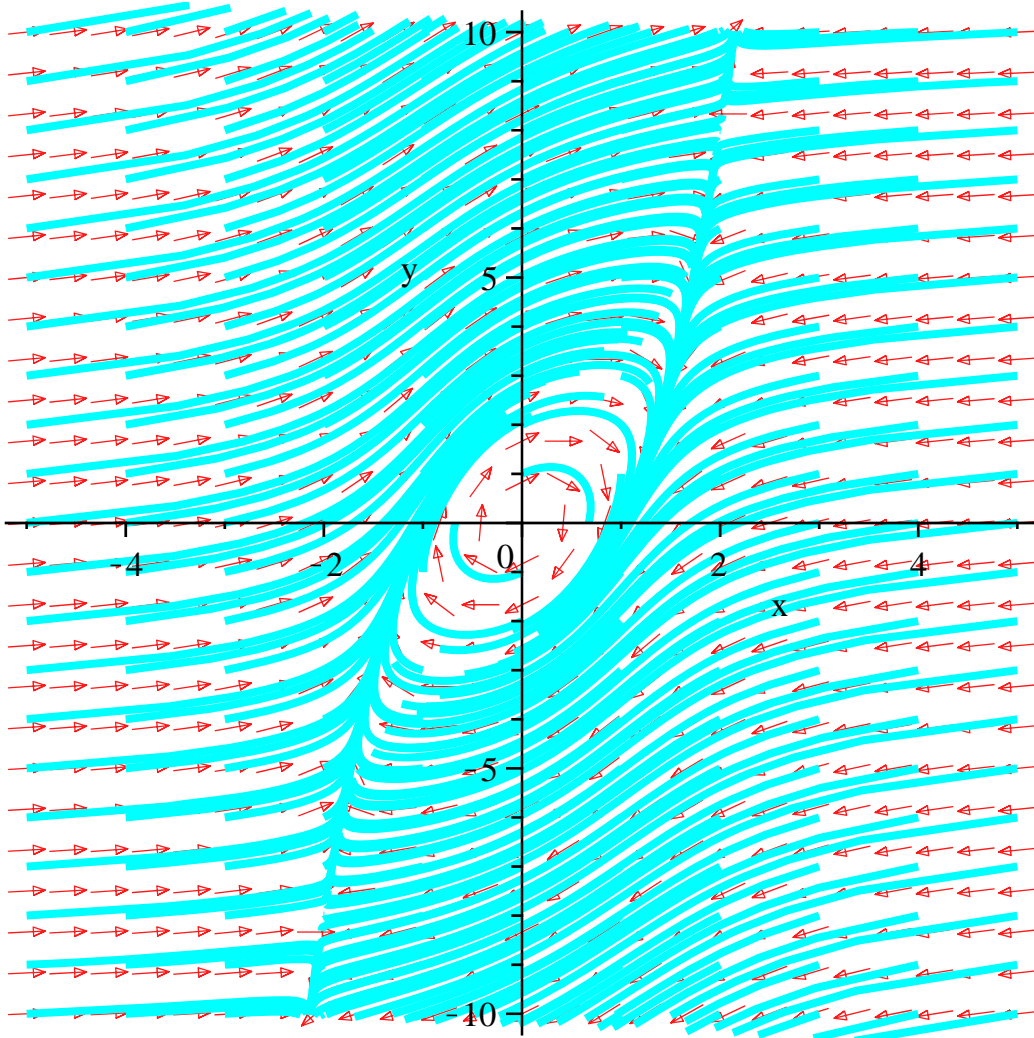
```
>
> DEplot( SYS2, [x(t),y(t)], t=0..1, x=-5..5, y=-10..10,
>   arrows=medium, dirgrid=[25,25], axes=boxed );
```



```

>
> IC := [ seq( seq( [0,a,b], b=[-10..10] ), a=[-5..5] ) ]:
>
> DEplot( SYS2, [x(t),y(t)], t=0..1, x=-5..5, y=-10..10, IC,
          arrows=medium, linecolor=cyan,
          dirgrid=[25,25], stepsize=0.05
          );

```



```
>
> EqSolns := solve( [f(x,y)=0,g(x,y)=0], [x,y] );
EqSolns := [[x=0,y=0], [x=2,y=8], [x=-2,y=-8]] (3.2)
```

```
>
> J := VectorCalculus:-Jacobian( [f(x,y),g(x,y)], [x,y] );
J := 
$$\begin{bmatrix} -3x^2 & 1 \\ -4 & 1 \end{bmatrix} (3.3)$$

```

```
> J1 := eval( J, EqSolns[1] );
J1 := 
$$\begin{bmatrix} 0 & 1 \\ -4 & 1 \end{bmatrix} (3.4)$$

```

```
> Eigenvalues( J1 ); (3.5)
```

$$\begin{bmatrix} \frac{1}{2} + \frac{1}{2} I\sqrt{15} \\ \frac{1}{2} - \frac{1}{2} I\sqrt{15} \end{bmatrix} \quad (3.5)$$

```
>
> J2 := eval( J, EqSolns[2] );
```

$$J2 := \begin{bmatrix} -12 & 1 \\ -4 & 1 \end{bmatrix} \quad (3.6)$$

```
> Eigenvalues( J2 );
```

$$\begin{bmatrix} -\frac{11}{2} + \frac{3}{2} \sqrt{17} \\ -\frac{11}{2} - \frac{3}{2} \sqrt{17} \end{bmatrix} \quad (3.7)$$

```
>
> J3 := eval( J, EqSolns[3] );
```

$$J3 := \begin{bmatrix} -12 & 1 \\ -4 & 1 \end{bmatrix} \quad (3.8)$$

```
> Eigenvalues( J3 );
```

$$\begin{bmatrix} -\frac{11}{2} + \frac{3}{2} \sqrt{17} \\ -\frac{11}{2} - \frac{3}{2} \sqrt{17} \end{bmatrix} \quad (3.9)$$

```
>
>
```