Math 300 Chapter 8 Homework Solutions

1.) Part a:

Statement S: The real number x^* is a maximizer of the function f if and only if for every real number x, $f(x) \leq f(x^*)$.

Not S: The real number x^* is not a maximizer of the function f if and only if there is a real number x such that $f(x) > f(x^*)$.

Part b:

Statement S: Suppose that f and g are functions of one variable. Then $g \geq f$ on the set S of real numbers if and only if for every element $x \in S$, $g(x) \geq f(x)$.

Not S: Suppose that f and g are functions of one variable. Then g is not greater than or equal to f on the set S of real numbers if and only if there is an element $x \in S$ such that $g(x) < f(x)$.

Part c:

Statement S: A real number is an upper bound for a set S of real numbers if and only if for all elements $x \in S$, $x \leq u$.

Not S: A real number u is not an upper bound for a set S of real numbers if and only if there is an element $x \in S$ such that $x > u$.

2.) Part a:

Statement S: A function f of one real variable is strictly increasing if and only if for all real numbers x and y with $x < y$, $f(x) < f(y)$.

Not S: A function f of one real variable is not strictly increasing if and only if there exists real numbers x and y with $x < y$ such that $f(x) \geq f(y)$.

Part b:

Statement S: The set C of real numbers is a convex set if and only if for all elements $x, y \in C$, and for every real number t with $0 \leq t \leq 1$, $tx + (1-t)y \in C$.

Not S: The set C of real numbers is not a convex set if and only if there exists elements $x, y \in C$, and there exists a real number t with $0 \leq t \leq 1$ such that $tx + (1-t)y \notin C$.

Part c:

Statement S: The function *f* of one real variable is a convex function if and only if for all real numbers x and y and for all real numbers t with $0 \leq t \leq 1$, it follows that $f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$.

Not S: The function *f* of one real variable is not a convex function if and only if there exists real numbers x and y and there is a real number t with $0 \leq t \leq 1$, such that $f(tx + (1-t)y) > tf(x) + (1-t)f(y)$.

3.) Part a:

A positive integer $p > 1$ is not prime if there is an integer n with $2 \leq n < p$ such that n divides p.

Part b:

A sequence $x_1, x_2, \ldots$ of real numbers is not increasing if there exists an integer k= 1, 2, … such that $x_k \geq x_{k+1}$.

Part c:

A sequence $x_1, x_2, \ldots$ of real numbers is not decreasing if there exists an integer k= 1, 2, …, $x_k \leq x_{k+1}$.

4.) Part a:

There does not exist an element $x \in S$ such that $x \in T$.

Part b:

For every angle t between 0 and 2π it is not the case that either $\sin(t) > \cos(t)$ or $\sin(t) < \cos(t)$.

5.) Part a:

The negation of the conclusion is: For all real numbers x, $x > a^{-x}$, or $x < a^{-x}$.

Part b:

The negation of the conclusion is: B.

6.) Part b:

(C or D) implies B

-Work forward from not B

-Work backward from (not C) and (not D)

Part d:

A implies (C or D)

-Work forward from (not C) and (not D)

-Work backward from not A

7.) Part a:

-Work forward from k divides n+1

-Work backward from: k is an integer that does divide an integer n

Part b:

-Work forward from mn is not divisible by 4 and n is divisible by 4

-Work backward from n is an odd integer or m is an even integer

10.) Part a:

Statement S: For every real number x > 0, $\sqrt{x} \leq x$.

Not S: There exists a real number x > 0, $\sqrt{x} > x$.

Now, use the construction method because of the "there exists" in the Not S. Construct $x = \frac{1}{4}$ so that $\sqrt{\frac{1}{4}} = \frac{1}{2} > \frac{1}{4}$ so that Not S is true.

Part b:

Statement S: For every positive integer n, $n^2 + n + 41$ is prime (the only positive integers to divide integer p where p > 1 are 1 and p).

Not S: There exists a positive integer n, where $n^2 + n + 41$ is not prime.

Now, use the construction method n= 41 so that $(41)^2 + 41 + 41 = 1763$ (which is divisible by 41 and 43).

Part c:

Statement S: If p is a positive integer that is not prime, then for every integer m with 1 < m ≤ $\sqrt{p}$, m does not divide p.

Not S: There exists an integer m with 1 < m ≤ $\sqrt{p}$ such that m divides the positive integer that is not prime p.

Now, use the construction method to make p= 100 and m= 10 so that 1 < 10 ≤ $\sqrt{100} = 10$ and 10 divides 100 (10k= 100 where integer k is 10).