7/9

# Chapter 11 HW

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1, 2a, 3, 4a, 5, 6, 7, 8, 9, 12

1. a) i) show that the lines $y=mx+b$ and $y=cx+d$ both pass through the points $(x_1,y_1)$ and $(x_2,y_2)$
 ii) conclude $y=mx+b$ and $y=cx+d$ are the same line
 b) i) show that $(x_1,y_1)$ and $(x_2,y_2)$ are solutions to the equations $ax+by=0$ and $cx+dy=0$
 ii) Conclude that $(x_1,y_1)=(x_2,y_2)$
 c) i) show that $(a+bi)(r+si)=(a+bi)(t+ui)=1$
 ii) conclude that $r+si=t+ui$
2. a) i) Show that $x^*$ and $y^*$ are both maximizers of the function $ax^2+bx+c$
 ii conclude that $x^*=y^*$
3. a) i) construct a line that goes through the 2 given points ($y=mx+b$). Then assume that $y=cx+d$ also passes through those 2 points. You must now work forward to show that these 2 lines are the same - that is that $m=c$ and $b=d$
 ii) First, construct a line, say $y=mx+b$, that goes through the 2 given points. Then assume that a different line $y=cx+d$ also passes through those 2 points. You must now work forward to reach a contradiction
 b) i) First construct a solution $(x_1,y_1)$ to the system of equations $ax+by=0$ & $cx+dy=0$. Then assume that you have another solution to the equations $(x_2,y_2)$ You must work

(ii) First construct a solution $(x_1, y_1)$ to the 2 equations $ax+by=0$ & $cx+dy=0$. then assume you have different solution $(x_2, y_2) \neq (x_1, y_1)$ To the 2 equations. You must work forward to get a contradiction

c) i) First construct a complex number $r+si$ that satisfys $(a+bi)(r+si)=1$. Then assume you have another complex number $t+ui$ that also satisfies $(a+bi)(r+si)=1$ you must work forward to show that $r+si$ and $t+ui$ are the same. $r+si = t+ui$

ii) First construct a complex number say $r+si$ that satisfies $(a+bi)(r+si)=1$ Then assume that there is a different complex number, say $t+ui \neq r+si$ that also satisfies $(a+bi)(r+si)=1$. You must work forward to reach a contradiction

4 a) i) obtain 2 qualifying objects $x^*$ ~~$\neq y^*$~~ and you must work to show $x^* = y^*$

ii) assume $x^* \neq y^*$ and we must work to obtain a contradiction

3/3

5. a) $u \in S$ with $x \le u$ for all $x \in S$
this is known to exist by the definition of the upper bound in the hypothesis

b) direct to work forward to show that $u = v$

c) specialization $v \in S$ $x \le v$ for all $x \in S$
specialize with $u = x$
$u \le v$

d) because when you specialize $x = v$ in $u \in S$ with $x \le u$ for all $x \in S$ and $x = u$ in $v \in S$ with $x \le v$ for all $x \in S$ we get $u = v$ so it is complete

1/3

6. maximizer: A real number $x^*$ such that for every real number $x$ $f(x) \le f(x^*)$

give an analysis of proof. method used? explain each step.

PROOF: by definition of a maximizer we know that a real number $x^*$ is a maximizer if for every real number $x$, $f(x) \le f(x^*)$. As proven in exercise 5.15. $x = 0$ is a maximizer of $f$. Suppose now that the real number $y$ is also a maximizer of $f$. so then by def. for every real number $z$, $f(z) \le f(y)$. There fore, $az^2 + c \le ay^2 + c$ We know by the hypothesis that $x = 0 \Rightarrow z = 0$ ? so $a(0)^2 + c \le ay^2 + c$ therefore $c \le ay^2 + c$. by Algebra we know $y^2 \le 0$. There fore $y = 0 = x$ so $x$ is the unique maximizer of the function $f(x) = ax^2 + c$.

direct method

specialized to $z=0$.

because $a<0$

7. By contradiction, assume $ad-bc=0$. Because $ax+by=0$ and $cx+dy=0$ have a unique solution, both $c$ and $d$ cannot be equal to 0. It then follows that $x=d$ and $y=-c$ is one solution to the 2 equations, so then $x=0$ and $y=0$ which means that $d=0$ and $c=0$ and since both $c$ and $d$ cannot be 0, there is a contradiction. Similarily $a$ and $b$ cannot be 0. It follows that $x=b$ and $y=-a$ so $x=0$ and $y=0$ so $a=0$ and $b=0$ which is a contradiction, so $ad-bc\neq 0$ so the proof is complete

8. A1: $y<0$ and $x=2y/(1+y)$
A2: $z$ is also a number with $z<0$ and $x=2z/(1+z)$
A3: $x=2y/(1+y)=2z/(1+z)$
A4: $y+yz=z+yz$

B1: $y=z$

PROOF: To show that $y$ is a unique solution suppose that $y$ and $z$ satisfy $y<0$, $z<0$, $x=2y/(1+y)$ and $x=2z/(1+z)$. Then we know that $2y/(1+y)=2z/(1+z)$ and so $y+yz=z+yz$ or $y=z$ so the proof is complete

9. $a \neq 0$ $a|b$ unique $k$ : $b = ka$

A1: $a|b$ $b = ka$
A2: let $m$ $a|b$ $b = ma$
A3: $b = ka$ & $b = ma$
A4: $ka = ma$
B1: $m = k$

3/3

We know that a divides b and $a \neq 0$. To show there is a unique solution suppose k and m satisfy $b = ka$ and $b = ma$. Then we know that $ka = ma$ and $k = m$. So k is a unique solution and the proof is complete

12. $(a+bi)(c+di)$

$ac + adi + bci - db$

$ac - db = 1$ $\qquad c = \frac{a}{a^2+b^2}$ $\qquad d = \frac{-b}{a^2+b^2}$

$bc + ad = 0$

$a^2+b^2 \neq 0$ by hyp

A1: $(a+bi)(c+di) = (ac - bd) + (bc + ad)i$

$= ((a^2+b^2)/(a^2+b^2)) + 0i$

$= 1$

A2: $(e+fi)(a+bi) = 1$

A3: $[(e+fi)(a+bi)](c+di) = e+fi$

B1: $c+di = e+fi$

<u>PROOF</u>: because either $a \neq$ or $b \neq 0$, $a^2+b^2 \neq 0$, and so we can construct the complex number $c+di$ in which $c = a/(a^2+b^2)$ and $d = -b/(a^2+b^2)$. Then $(a+bi)(c+di) = (ac-bd) + (bc-ad)i = 1$. To see that this is unique assume $e+fi$ also satisfies $(a+bi)(e+fi) = 1$. Multiplying through by $e+fi$ yields $[(e+fi)(a+bi)](c+di) = e+fi$. Using $(a+bi)(e+fi) = 1$, it follows that $c+di = e+fi$ so it is unique. □