```
> restart;
  readlib( laplace ):
  with( student ):
```

Homework Solutions for Section 9.4

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Math 242

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Problem #8

This is the same as Example 2. except that two of the flow rates are increased. The new set of ODEs is

```
> ODE1 := diff( x(t), t ) = 12 + 3/100*y(t) - 5/100*x(t) - 4/100*x(t);
```

$$ODE1 := \frac{\partial}{\partial t}\mathrm{x}(t) = 12 + \frac{3}{100}\mathrm{y}(t) - \frac{9}{100}\mathrm{x}(t)$$

```
> ODE2 := diff( y(t), t ) = 5/100*x(t) -5/100*y(t);
```

$$ODE2 := \frac{\partial}{\partial t}\mathrm{y}(t) = \frac{1}{20}\mathrm{x}(t) - \frac{1}{20}\mathrm{y}(t)$$

```
> IC1 := x(0) = 0;
```

$$IC1 := \mathrm{x}(0) = 0$$

```
> IC2 := y(0) = 200;
```

$$IC2 := \mathrm{y}(0) = 200$$

We could go through the Laplace Transform procedure (as in Section 9.3), but in the interest of efficiency, I'll simply use dsolve

```
> SOLN := dsolve( { ODE1, ODE2, IC1, IC2 }, { x(t), y(t) } );
```

$$SOLN := \Bigg\{ \mathrm{x}(t) = \Bigg($$

$$-6000 - 30\left(\frac{200}{19}\sqrt{19} - 100\right)e^{\left(1/100\, t\left(\sqrt{19} - 7\right)\right)} - 30\left(-\frac{200}{19}\sqrt{19} - 100\right)e^{\left(-1/100\, t\left(\sqrt{19} + 7\right)\right)}\Bigg) \Big/ \Big($$

$$\left(\sqrt{19} - 7\right)\left(\sqrt{19} + 7\right)\Big), \mathrm{y}(t) = \Bigg(-6000$$

$$-30\left(\frac{1}{3}\left(\frac{200}{19}\sqrt{19} - 100\right)\sqrt{19} + \frac{400}{57}\sqrt{19} - \frac{200}{3}\right)e^{\left(1/100\, t\left(\sqrt{19} - 7\right)\right)}$$

$$- \frac{15000}{19} \sqrt{19}\, e^{\left(- 1/100\, t\, \left(\sqrt{19} + 7\right)\right)} \Bigg) \Big/ \left(\left(\sqrt{19} - 7\right) \left(\sqrt{19} + 7\right) \right) \Bigg\}$$

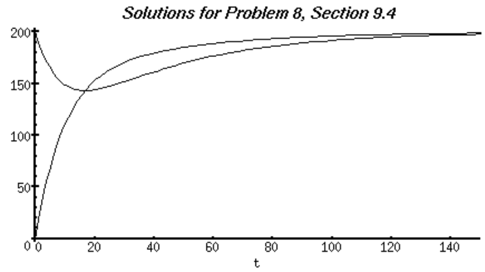
The constants in this solution are more than a little messy. Here's an approximate solution with floating point coefficients.

> evalf(");

$$\{ x(t) = 200.0000001 - 54.11685326\, e^{(-.02641101056\, t)} - 145.8831469\, e^{(-.1135889894\, t)},$$
$$y(t) = 200.0000001 - 114.7078670\, e^{(-.02641101056\, t)} + 114.7078670\, e^{(-.1135889894\, t)} \}$$

Even now the solutions are difficult to visualize. Fortunately, Maple will quickly plot the solutions.

> plot(map(rhs, SOLN), t=0..150, title=‘Solutions for Problem 8, Section 9.4‘);



>

Problem #9

The diagram of the two tanks in the book leads to the two differential equations

> ODE1 := diff(x(t), t) = 15 - 5/50*x(t);

$$ODE1 := \frac{\partial}{\partial t}\,\mathrm{x}(t) = 15 - \frac{1}{10}\,\mathrm{x}(t)$$

> ODE2 := diff(y(t), t) = 5/50*x(t) -5/50*y(t);

$$ODE2 := \frac{\partial}{\partial t}\,\mathrm{y}(t) = \frac{1}{10}\,\mathrm{x}(t) - \frac{1}{10}\,\mathrm{y}(t)$$

and initial conditions

> IC1 := x(0) = 50;

$$IC1 := \mathrm{x}(0) = 50$$

> IC2 := y(0) = 100;

$$IC2 := \mathrm{y}(0) = 100$$

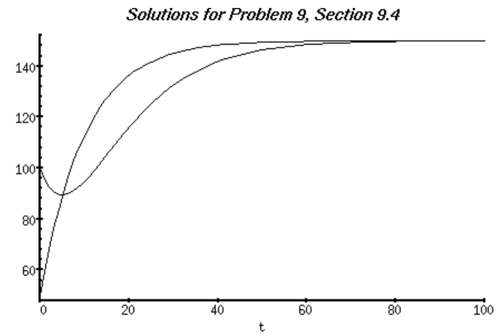
The solution to this IVP is

> SOLN := dsolve({ ODE1, ODE2, IC1, IC2 }, { x(t), y(t) });

$$SOLN := \{\, \mathrm{y}(t) = 150 - 50\,\mathrm{e}^{(-1/10\,t)} - 10\,\mathrm{e}^{(-1/10\,t)}\,t,\ \mathrm{x}(t) = 150 - 100\,\mathrm{e}^{(-1/10\,t)} \,\}$$

To conclude, let's plot the two masses as a function of time

> plot(map(rhs, SOLN), t=0..100, title='Solutions for Problem 9, Section 9.4');



>

Problem #10

```
> ODE1 := diff( x(t), t ) = 8 + 3/100*y(t) - 1/100*x(t) - 4/100*x(t);
```

$$ODE1 := \frac{\partial}{\partial t}\, \mathrm{x}(t) = 8 + \frac{3}{100}\, \mathrm{y}(t) - \frac{1}{20}\, \mathrm{x}(t)$$

```
> ODE2 := diff( y(t), t ) = 1/100*x(t) -3/100*y(t);
```

$$ODE2 := \frac{\partial}{\partial t}\, \mathrm{y}(t) = \frac{1}{100}\, \mathrm{x}(t) - \frac{3}{100}\, \mathrm{y}(t)$$

```
> IC1 := x(0) = 0;
```

$$IC1 := \mathrm{x}(0) = 0$$

```
> IC2 := y(0) = 200;
```

$$IC2 := \mathrm{y}(0) = 200$$

```
>
```

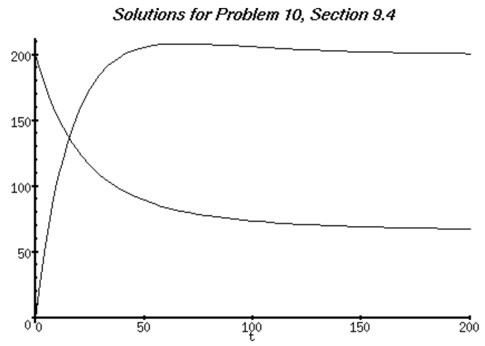
In this case the solution is found to be

```
> SOLN := dsolve( { ODE1, ODE2, IC1, IC2 }, { x(t), y(t) } );
```

$$SOLN := \left\{ \mathrm{y}(t) = \frac{200}{3} + 50\, \mathrm{e}^{(-1/50\, t)} + \frac{250}{3}\, \mathrm{e}^{(-3/50\, t)},\ \mathrm{x}(t) = 200 + 50\, \mathrm{e}^{(-1/50\, t)} - 250\, \mathrm{e}^{(-3/50\, t)} \right\}$$

Here is what those solutions might look like.

```
> plot( map( rhs, SOLN), t=0..200, title=‘Solutions for Problem 10, Section 9.4‘ );
```




```
>
```

END OF WORKSHEET

```
>
```