```
> restart;
  readlib( laplace ):
  with( student ):
```

Homework Solutions for Section 9.3

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Math 242

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Problem #1

```
> ODE1 := diff( x(t), t ) = 3*x(t) - 2*y(t);
```

$$ODE1 := \frac{\partial}{\partial t}\,\mathrm{x}(t) = 3\,\mathrm{x}(t) - 2\,\mathrm{y}(t)$$

```
> ODE2 := diff( y(t), t ) = 3*y(t) - 2*x(t);
```

$$ODE2 := \frac{\partial}{\partial t}\,\mathrm{y}(t) = 3\,\mathrm{y}(t) - 2\,\mathrm{x}(t)$$

```
> IC1 := x(0)=1;
```

$$IC1 := \mathrm{x}(0) = 1$$

```
> IC2 := y(0) = 1;
```

$$IC2 := \mathrm{y}(0) = 1$$

```
> lapSYS := laplace( {ODE1,ODE2}, t, s );
```

$$lapSYS := \{\mathrm{laplace}(\mathrm{y}(t), t, s)\,s - \mathrm{y}(0) = 3\,\mathrm{laplace}(\mathrm{y}(t), t, s) - 2\,\mathrm{laplace}(\mathrm{x}(t), t, s),$$
$$\mathrm{laplace}(\mathrm{x}(t), t, s)\,s - \mathrm{x}(0) = 3\,\mathrm{laplace}(\mathrm{x}(t), t, s) - 2\,\mathrm{laplace}(\mathrm{y}(t), t, s)\}$$

```
> lapSYS := subs( { IC1, IC2 }, lapSYS );
```

$$lapSYS := \{\mathrm{laplace}(\mathrm{x}(t), t, s)\,s - 1 = 3\,\mathrm{laplace}(\mathrm{x}(t), t, s) - 2\,\mathrm{laplace}(\mathrm{y}(t), t, s),$$
$$\mathrm{laplace}(\mathrm{y}(t), t, s)\,s - 1 = 3\,\mathrm{laplace}(\mathrm{y}(t), t, s) - 2\,\mathrm{laplace}(\mathrm{x}(t), t, s)\}$$

```
> lapSOLN := solve( lapSYS, { laplace( y(t), t, s ), laplace( x(t), t, s ) } );
```

$$lapSOLN := \left\{\mathrm{laplace}(\mathrm{y}(t), t, s) = \frac{1}{s-1}, \mathrm{laplace}(\mathrm{x}(t), t, s) = \frac{1}{s-1}\right\}$$

```
>
```

The solution to the system is now seen to be
x(t) = exp(t)
y(t) = exp(t)

Problem #3

```
> ODE1 := diff( z(t), t ) + 2*w(t) = 0;
```

$$ODE1 := \left(\frac{\partial}{\partial t} \mathrm{z}(t)\right) + 2\,\mathrm{w}(t) = 0$$

```
> ODE2 := diff( z(t), t ) - diff( w(t), t ) = 0;
```

$$ODE2 := \left(\frac{\partial}{\partial t} \mathrm{z}(t)\right) - \left(\frac{\partial}{\partial t} \mathrm{w}(t)\right) = 0$$

```
> IC1 := z(0) = -1;
```

$$IC1 := \mathrm{z}(0) = -1$$

```
> IC2 := w(0) = 2;
```

$$IC2 := \mathrm{w}(0) = 2$$

```
> lapSYS := laplace( {ODE1,ODE2}, t, s );
```

$$lapSYS := \{\mathrm{laplace}(\mathrm{z}(t), t, s)\, s - \mathrm{z}(0) + 2\,\mathrm{laplace}(\mathrm{w}(t), t, s) = 0,$$
$$\mathrm{laplace}(\mathrm{z}(t), t, s)\, s - \mathrm{z}(0) - \mathrm{laplace}(\mathrm{w}(t), t, s)\, s + \mathrm{w}(0) = 0\}$$

```
> lapSYS := subs( { IC1, IC2 }, lapSYS );
```

$$lapSYS := \{\mathrm{laplace}(\mathrm{z}(t), t, s)\, s + 1 + 2\,\mathrm{laplace}(\mathrm{w}(t), t, s) = 0,$$
$$\mathrm{laplace}(\mathrm{z}(t), t, s)\, s + 3 - \mathrm{laplace}(\mathrm{w}(t), t, s)\, s = 0\}$$

```
> lapSOLN := solve( lapSYS, { laplace( z(t), t, s ), laplace( w(t), t, s ) } );
```

$$lapSOLN := \left\{\mathrm{laplace}(\mathrm{z}(t), t, s) = -\frac{s+6}{s\,(s+2)}, \mathrm{laplace}(\mathrm{w}(t), t, s) = 2\,\frac{1}{s+2}\right\}$$

```
> convert( lapSOLN, parfrac, s );
```

$$\left\{\mathrm{laplace}(\mathrm{z}(t), t, s) = -3\,\frac{1}{s} + 2\,\frac{1}{s+2}, \mathrm{laplace}(\mathrm{w}(t), t, s) = 2\,\frac{1}{s+2}\right\}$$

```
>
```

The inverse Laplace transform of this is seen to be

*z(t) = -3 + 2*exp(-2*t)*
*w(t) = 2*exp(-2*t)*

```
>
```

Problem #6

> ODE1 := diff(x(t), t) - x(t) - y(t) = 1;

$$ODE1 := \left(\frac{\partial}{\partial t}\,\mathrm{x}(t)\right) - \mathrm{x}(t) - \mathrm{y}(t) = 1$$

> ODE2 := -x(t) + diff(y(t), t) - y(t) = 0;

$$ODE2 := -\mathrm{x}(t) + \left(\frac{\partial}{\partial t}\,\mathrm{y}(t)\right) - \mathrm{y}(t) = 0$$

> IC1 := x(0) = 0;

$$IC1 := \mathrm{x}(0) = 0$$

> IC2 := y(0) = -5/2;

$$IC2 := \mathrm{y}(0) = \frac{-5}{2}$$

> lapSYS := laplace({ODE1,ODE2}, t, s);

$$lapSYS := \{-\mathrm{laplace}(\mathrm{x}(t), t, s) + \mathrm{laplace}(\mathrm{y}(t), t, s)\, s - \mathrm{y}(0) - \mathrm{laplace}(\mathrm{y}(t), t, s) = 0,$$
$$\mathrm{laplace}(\mathrm{x}(t), t, s)\, s - \mathrm{x}(0) - \mathrm{laplace}(\mathrm{x}(t), t, s) - \mathrm{laplace}(\mathrm{y}(t), t, s) = \frac{1}{s}\}$$

> lapSYS := subs({ IC1, IC2 }, lapSYS);

$$lapSYS := \{\mathrm{laplace}(\mathrm{x}(t), t, s)\, s - \mathrm{laplace}(\mathrm{x}(t), t, s) - \mathrm{laplace}(\mathrm{y}(t), t, s) = \frac{1}{s},$$
$$-\mathrm{laplace}(\mathrm{x}(t), t, s) + \mathrm{laplace}(\mathrm{y}(t), t, s)\, s + \frac{5}{2} - \mathrm{laplace}(\mathrm{y}(t), t, s) = 0\}$$

> lapSOLN := solve(lapSYS, { laplace(y(t), t, s), laplace(x(t), t, s) });

$$lapSOLN := \left\{\mathrm{laplace}(\mathrm{y}(t), t, s) = -\frac{1}{2}\,\frac{-2 + 5\,s^2 - 5\,s}{s^2\,(s-2)},\ \mathrm{laplace}(\mathrm{x}(t), t, s) = -\frac{1}{2}\,\frac{3\,s + 2}{s^2\,(s-2)}\right\}$$

> convert(lapSOLN, parfrac, s);

$$\left\{\mathrm{laplace}(\mathrm{x}(t), t, s) = \frac{1}{2}\,\frac{1}{s^2} + \frac{1}{s} - \frac{1}{s-2},\ \mathrm{laplace}(\mathrm{y}(t), t, s) = -\frac{1}{2}\,\frac{1}{s^2} - \frac{3}{2}\,\frac{1}{s} - \frac{1}{s-2}\right\}$$

>

The solution is therefore seen to be
*x(t) = 1 + 0.5*t - exp(2*t)*
*y(t) = -1.5 - 0.5*t - exp(2*t)*

>

Problem #10

```
> ODE1 := diff( x(t), t$2 ) + y(t) = 1;
```

$$ODE1 := \left(\frac{\partial^2}{\partial t^2}\,\mathrm{x}(t)\right) + \mathrm{y}(t) = 1$$

```
> ODE2 := x(t) + diff( y(t), t$2 ) = -1;
```

$$ODE2 := \mathrm{x}(t) + \left(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)\right) = -1$$

```
> IC1 := x(0) = 1, D(x)(0) = 1;
```

$$IC1 := \mathrm{x}(0) = 1,\ \mathrm{D}(x)(0) = 1$$

```
> IC2 := y(0) = 1, D(y)(0) = -1;
```

$$IC2 := \mathrm{y}(0) = 1,\ \mathrm{D}(y)(0) = -1$$

```
> lapSYS := laplace( {ODE1,ODE2}, t, s );
```

$$lapSYS := \Big\{\mathrm{laplace}(\mathrm{x}(t), t, s) + (\mathrm{laplace}(\mathrm{y}(t), t, s)\, s - \mathrm{y}(0))\, s - \mathrm{D}(y)(0) = -\frac{1}{s},$$
$$(\mathrm{laplace}(\mathrm{x}(t), t, s)\, s - \mathrm{x}(0))\, s - \mathrm{D}(x)(0) + \mathrm{laplace}(\mathrm{y}(t), t, s) = \frac{1}{s}\Big\}$$

```
> lapSYS := subs( { IC1, IC2 }, lapSYS );
```

$$lapSYS := \Big\{(\mathrm{laplace}(\mathrm{x}(t), t, s)\, s - 1)\, s - 1 + \mathrm{laplace}(\mathrm{y}(t), t, s) = \frac{1}{s},$$
$$\mathrm{laplace}(\mathrm{x}(t), t, s) + (\mathrm{laplace}(\mathrm{y}(t), t, s)\, s - 1)\, s + 1 = -\frac{1}{s}\Big\}$$

```
> lapSOLN := solve( lapSYS, { laplace( y(t), t, s ), laplace( x(t), t, s ) } );
```

$$lapSOLN := \left\{\mathrm{laplace}(\mathrm{x}(t), t, s) = \frac{s^3 + 1}{s\,(s^3 - s^2 + s - 1)},\ \mathrm{laplace}(\mathrm{y}(t), t, s) = \frac{s^3 - 2\,s^2 - 1}{s\,(s^3 - s^2 + s - 1)}\right\}$$

```
> convert( lapSOLN, parfrac, s );
```

$$\left\{\mathrm{laplace}(\mathrm{x}(t), t, s) = -\frac{1}{s} + \frac{1}{s-1} + \frac{s}{s^2+1},\ \mathrm{laplace}(\mathrm{y}(t), t, s) = \frac{1}{s} - \frac{1}{s-1} + \frac{s}{s^2+1}\right\}$$

```
>
```

In this form the inverse Laplace transform can be applied with little difficulty. The solution is

x(t) = 1 + exp(t) + cos(t)
y(t) = 1 - exp(t) + cos(t)

Problem #15

```
> ODE1 := diff( x(t), t ) = 3*x(t) + y(t) - 2*z(t);
```

$$ODE1 := \frac{\partial}{\partial t}\,\mathrm{x}(t) = 3\,\mathrm{x}(t) + \mathrm{y}(t) - 2\,\mathrm{z}(t)$$

```
> ODE2 := diff( y(t), t ) = -x(t) + 2*y(t) + z(t);
```

$$ODE2 := \frac{\partial}{\partial t}\,\mathrm{y}(t) = -\mathrm{x}(t) + 2\,\mathrm{y}(t) + \mathrm{z}(t)$$

```
> ODE3 := diff( z(t), t ) = 4*x(t) + y(t) - 3*x(t);
```

$$ODE3 := \frac{\partial}{\partial t}\,\mathrm{z}(t) = \mathrm{x}(t) + \mathrm{y}(t)$$

```
> IC1 := x(0)=-6;
```

$$IC1 := \mathrm{x}(0) = -6$$

```
> IC2 := y(0) = 2;
```

$$IC2 := \mathrm{y}(0) = 2$$

```
> IC3 := z(0) = -12;
```

$$IC3 := \mathrm{z}(0) = -12$$

```
> lapSYS := laplace( {ODE1,ODE2, ODE3}, t, s );
```

$$\begin{aligned}lapSYS := \{&\mathrm{laplace}(\mathrm{y}(t), t, s)\, s - \mathrm{y}(0) = \\ &-\mathrm{laplace}(\mathrm{x}(t), t, s) + 2\,\mathrm{laplace}(\mathrm{y}(t), t, s) + \mathrm{laplace}(\mathrm{z}(t), t, s), \\ &\mathrm{laplace}(\mathrm{z}(t), t, s)\, s - \mathrm{z}(0) = \mathrm{laplace}(\mathrm{x}(t), t, s) + \mathrm{laplace}(\mathrm{y}(t), t, s), \\ &\mathrm{laplace}(\mathrm{x}(t), t, s)\, s - \mathrm{x}(0) = \\ &3\,\mathrm{laplace}(\mathrm{x}(t), t, s) + \mathrm{laplace}(\mathrm{y}(t), t, s) - 2\,\mathrm{laplace}(\mathrm{z}(t), t, s)\}\end{aligned}$$

```
> lapSYS := subs( { IC1, IC2, IC3 }, lapSYS );
```

$$\begin{aligned}lapSYS := \{&\mathrm{laplace}(\mathrm{z}(t), t, s)\, s + 12 = \mathrm{laplace}(\mathrm{x}(t), t, s) + \mathrm{laplace}(\mathrm{y}(t), t, s), \\ &\mathrm{laplace}(\mathrm{y}(t), t, s)\, s - 2 = -\mathrm{laplace}(\mathrm{x}(t), t, s) + 2\,\mathrm{laplace}(\mathrm{y}(t), t, s) + \mathrm{laplace}(\mathrm{z}(t), t, s), \\ &\mathrm{laplace}(\mathrm{x}(t), t, s)\, s + 6 = 3\,\mathrm{laplace}(\mathrm{x}(t), t, s) + \mathrm{laplace}(\mathrm{y}(t), t, s) - 2\,\mathrm{laplace}(\mathrm{z}(t), t, s)\}\end{aligned}$$

```
> lapSOLN := solve( lapSYS, { laplace( y(t), t, s ), laplace( x(t), t, s ), laplace( z(t),
  t, s ) } );
```

$$\begin{aligned}lapSOLN := \Big\{&\mathrm{laplace}(\mathrm{z}(t), t, s) = -2\,\frac{6\,s^2 - 28\,s + 35}{(s-1)\,(-4\,s + 4 + s^2)}, \\ &\mathrm{laplace}(\mathrm{x}(t), t, s) = -2\,\frac{3\,s^2 - 19\,s + 29}{(s-1)\,(-4\,s + 4 + s^2)}, \mathrm{laplace}(\mathrm{y}(t), t, s) = 2\,\frac{s-5}{-4\,s + 4 + s^2}\Big\}\end{aligned}$$

```
> convert( lapSOLN, parfrac, s );
```

$$\left\{\mathrm{laplace}(\mathrm{y}(t), t, s) = -6\,\frac{1}{(s-2)^2} + 2\,\frac{1}{s-2},\right.$$

$$\mathrm{laplace}(\mathrm{x}(t), t, s) = -26\,\frac{1}{s-1} - 6\,\frac{1}{(s-2)^2} + 20\,\frac{1}{s-2},$$

$$\left.\mathrm{laplace}(\mathrm{z}(t), t, s) = -26\,\frac{1}{s-1} - 6\,\frac{1}{(s-2)^2} + 14\,\frac{1}{s-2}\right\}$$

>

The solution can now be obtained with the use of the Table. The three components of the solution are

*x(t) = -26*exp(t) - 6*t*exp(2*t) + 20*exp(2*t)*
*y(t) = -6*t*exp(2*t) + 2*exp(2*t)*
*z(t) = -26*exp(t) - 6*t*exp(2*t) + 14*exp(2*t)*

>

END OF WORKSHEET

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