```
> restart;
  readlib( laplace ):
  with( student ):
```

Homework Solutions for Section 7.5

Prepared by Doug Meade
Math 242

1 May 1995

Problem #3

```
> ODE := diff( y(t), t$2 ) + 6*diff( y(t), t ) + 9*y(t) = 0;
```

$$ODE := \left(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)\right) + 6\left(\frac{\partial}{\partial t}\,\mathrm{y}(t)\right) + 9\,\mathrm{y}(t) = 0$$

```
> IC := y(0)=-1, D(y)(0)=6;
```

$$IC := \mathrm{y}(0) = -1,\ \mathrm{D}(y)(0) = 6$$

```
> lapODE := laplace( ODE, t, s );
```

$$lapODE := (\mathrm{laplace}(\mathrm{y}(t), t, s)\,s - \mathrm{y}(0))\,s - \mathrm{D}(y)(0) + 6\,\mathrm{laplace}(\mathrm{y}(t), t, s)\,s - 6\,\mathrm{y}(0) + 9\,\mathrm{laplace}(\mathrm{y}(t), t, s) = 0$$

```
> lapODE := subs( IC, lapODE );
```

$$lapODE := (\mathrm{laplace}(\mathrm{y}(t), t, s)\,s + 1)\,s + 6\,\mathrm{laplace}(\mathrm{y}(t), t, s)\,s + 9\,\mathrm{laplace}(\mathrm{y}(t), t, s) = 0$$

```
> Y := solve( lapODE, laplace( y(t), t, s ) );
```

$$Y := -\frac{s}{s^2 + 6s + 9}$$

```
> Y := convert( Y, parfrac, s );
```

$$Y := 3\,\frac{1}{(s+3)^2} - \frac{1}{s+3}$$

```
>
```

Using the Table to help with the inverse Laplace transform, the solution is now seen to be

*y(t) = 3*t*exp(-3*t) - exp(-3*t)*

```
>
```

Problem #4

> ODE := diff(y(t), t$2) + 6*diff(y(t), t) + 5*y(t) = 12*exp(t);

$$ODE := \left(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)\right) + 6\left(\frac{\partial}{\partial t}\,\mathrm{y}(t)\right) + 5\,\mathrm{y}(t) = 12\,\mathrm{e}^{t}$$

> IC := y(0)=-1, D(y)(0)=7;

$$IC := \mathrm{y}(0) = -1,\ \mathrm{D}(y)(0) = 7$$

> lapODE := laplace(ODE, t, s);

$$lapODE := (\mathrm{laplace}(\mathrm{y}(t), t, s)\,s - \mathrm{y}(0))\,s - \mathrm{D}(y)(0) + 6\,\mathrm{laplace}(\mathrm{y}(t), t, s)\,s - 6\,\mathrm{y}(0) + 5\,\mathrm{laplace}(\mathrm{y}(t), t, s) = 12\,\frac{1}{s-1}$$

> lapODE := subs(IC, lapODE);

lapODE :=

$$(\mathrm{laplace}(\mathrm{y}(t), t, s)\,s + 1)\,s - 1 + 6\,\mathrm{laplace}(\mathrm{y}(t), t, s)\,s + 5\,\mathrm{laplace}(\mathrm{y}(t), t, s) = 12\,\frac{1}{s-1}$$

> Y := solve(lapODE, laplace(y(t), t, s));

$$Y := -\frac{s - 1 - 12\,\frac{1}{s-1}}{s^2 + 6\,s + 5}$$

> Y := convert(Y, parfrac, s);

$$Y := \frac{1}{s-1} - \frac{1}{s+5} - \frac{1}{s+1}$$

>

The inverse Laplace transform is

*y(t) = exp(t) - exp(-5*t) - exp(-t)*

>

Problem #17

> ODE := diff(y(t), t$2) + diff(y(t), t) - y(t) = t^3;

$$ODE := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + \left(\frac{\partial}{\partial t} y(t)\right) - y(t) = t^3$$

> IC := y(0)=-1, D(y)(0)=0;

$$IC := y(0) = -1, D(y)(0) = 0$$

> lapODE := laplace(ODE, t, s);

$$lapODE := (\text{laplace}(y(t), t, s)\, s - y(0))\, s - D(y)(0) + \text{laplace}(y(t), t, s)\, s - y(0) - \text{laplace}(y(t), t, s) = 6\frac{1}{s^4}$$

> lapODE := subs(IC, lapODE);

$$lapODE := (\text{laplace}(y(t), t, s)\, s + 1)\, s + \text{laplace}(y(t), t, s)\, s + 1 - \text{laplace}(y(t), t, s) = 6\frac{1}{s^4}$$

> Y := solve(lapODE, laplace(y(t), t, s));

$$Y := -\frac{s + 1 - 6\frac{1}{s^4}}{s^2 + s - 1}$$

The Laplace transform of the solution is, after simpilfication of the complex fraction,

> simplify(Y);

$$-\frac{s^5 + s^4 - 6}{s^4 (s^2 + s - 1)}$$

>

Problem #20

> ODE := diff(y(t), t$2) + 3*diff(y(t), t) = t^3;

$$ODE := \left(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)\right) + 3\left(\frac{\partial}{\partial t}\,\mathrm{y}(t)\right) = t^3$$

> IC := y(0)=0, D(y)(0)=0;

$$IC := \mathrm{y}(0) = 0,\ \mathrm{D}(y)(0) = 0$$

> lapODE := laplace(ODE, t, s);

$$lapODE := (\mathrm{laplace}(\mathrm{y}(t), t, s)\, s - \mathrm{y}(0))\, s - \mathrm{D}(y)(0) + 3\,\mathrm{laplace}(\mathrm{y}(t), t, s)\, s - 3\,\mathrm{y}(0) = 6\,\frac{1}{s^4}$$

> lapODE := subs(IC, lapODE);

$$lapODE := \mathrm{laplace}(\mathrm{y}(t), t, s)\, s^2 + 3\,\mathrm{laplace}(\mathrm{y}(t), t, s)\, s = 6\,\frac{1}{s^4}$$

> Y := solve(lapODE, laplace(y(t), t, s));

$$Y := 6\,\frac{1}{s^4\,(s^2 + 3\,s)}$$

Maple would have no trouble finding the partial fraction decomposition, but it's not required of the present problem.

>

Problem #30

>

> ODE := diff(y(t), t$2) + 6*diff(y(t), t) + 5*y(t) = t;

$$ODE := \left(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)\right) + 6\left(\frac{\partial}{\partial t}\,\mathrm{y}(t)\right) + 5\,\mathrm{y}(t) = t$$

> IC := y(0)=a, D(y)(0)=b;

$$IC := \mathrm{y}(0) = a,\ \mathrm{D}(y)(0) = b$$

> lapODE := laplace(ODE, t, s);

$$lapODE := (\mathrm{laplace}(\mathrm{y}(t), t, s)\,s - \mathrm{y}(0))\,s - \mathrm{D}(y)(0) + 6\,\mathrm{laplace}(\mathrm{y}(t), t, s)\,s - 6\,\mathrm{y}(0) + 5\,\mathrm{laplace}(\mathrm{y}(t), t, s) = \frac{1}{s^2}$$

> lapODE := subs(IC, lapODE);

lapODE :=

$$(\mathrm{laplace}(\mathrm{y}(t), t, s)\,s - a)\,s - b + 6\,\mathrm{laplace}(\mathrm{y}(t), t, s)\,s - 6\,a + 5\,\mathrm{laplace}(\mathrm{y}(t), t, s) = \frac{1}{s^2}$$

> Y := solve(lapODE, laplace(y(t), t, s));

$$Y := -\frac{-s\,a - b - 6\,a - \frac{1}{s^2}}{s^2 + 6\,s + 5}$$

> Y := convert(Y, parfrac, s);

$$Y := -\frac{6}{25}\frac{1}{s} + \frac{1}{5}\frac{1}{s^2} - \frac{1}{100}\frac{25\,a + 25\,b + 1}{s + 5} + \frac{1}{4}\frac{5\,a + b + 1}{s + 1}$$

>

Thus, the solution to the IVP is seen to be

*y(t) = -0.24 + 0.2*t - (25*(a+b)+1)/100*exp(-5*t) + (5*a+b+1)/4*exp(-t)*

>

END OF WORKSHEET

>