```
> restart;
  with( DEtools ): with( plots ):
```

Maple Worksheet to Accompany the Solution to Problem # 13 (Section 3.4)

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Filename: soln3-4-13.ms

As described on the written part of the solution, the ODE for the upward motion is

```
> ode := 3*diff( v(t), t ) = 3*981/100 + 1/10*v(t)^2;
```

$$ode := 3\left(\frac{\partial}{\partial t}\,\mathrm{v}(t)\right) = \frac{2943}{100} + \frac{1}{10}\,\mathrm{v}(t)^2$$

with initial condition

```
> ic := v(0)=-500;
```

$$ic := \mathrm{v}(0) = -500$$

The velocity and displacement are then found to be

```
> vel := rhs( dsolve( { ode, ic }, v(t) ) );
```

$$vel := -\frac{3}{10}\tan\left(\frac{1}{100}\left(-t + \frac{10}{327}\arctan\left(\frac{500}{981}\sqrt{3270}\right)\sqrt{3270}\right)\sqrt{3270}\right)\sqrt{3270}$$

```
> dis := rhs( dsolve( { diff( x(t), t ) = vel, x(0)=0 }, x(t) ) );
```

$$dis := -30\ln\left(\cos\left(\frac{1}{100}\sqrt{3270}\,t\right)\cos\left(\arctan\left(\frac{500}{981}\sqrt{3270}\right)\right) + \sin\left(\frac{1}{100}\sqrt{3270}\,t\right)\sin\left(\arctan\left(\frac{500}{981}\sqrt{3270}\right)\right)\right)$$
$$+ 30\ln\left(\frac{3}{2502943}\sqrt{818462361}\right)$$

Note that these expressions are EXACT but rather difficult to read. Approximate solutions which show the basic form of the solution can be obtained using EVALF.

```
> evalf( vel );
```

$$-17.15517415\tan(-.5718391382\,t + 1.536499432)$$

```
> evalf( dis );
```

$$-30.\ln(.03429017104\cos(.5718391382\,t) + .9994119192\sin(.5718391382\,t)) - 101.1868957$$

The maximum height occurs when the shell has no velocity

> Tmax := solve(vel=0, t);

$$Tmax := \frac{10}{327}\arctan\left(\frac{500}{981}\sqrt{3270}\right)\sqrt{3270}$$

> evalf(Tmax);

2.686943459

The height at which this occurs is the displacement evaluated at the above time

> Hmax := subs(t=Tmax, dis);

$$Hmax := -30\ln\left(\frac{3}{2502943}\cos\left(\arctan\left(\frac{500}{981}\sqrt{3270}\right)\right)\sqrt{818462361}\right.$$
$$\left.+\frac{500}{818462361}\sin\left(\arctan\left(\frac{500}{981}\sqrt{3270}\right)\right)\sqrt{3270}\sqrt{818462361}\right)+30\ln\left(\frac{3}{2502943}\sqrt{818462361}\right)$$

> evalf(Hmax);

-101.1868957

So the maximum height of ~101.187 meters is reached approximately 2.687 seconds after launch.

To emphasize that all calculations have been done using EXACT answers (the floating point answers are used only for our visual benefit). I will now repeat the final calculations using approximate solutions.

> velAPPROX := -17.16*tan(-0.57*t+1.54);

$$velAPPROX := 17.16\tan(.57\,t - 1.54)$$

> disAPPROX := -30.*ln(0.03*cos(0.57*t)+.99*sin(0.57*t))-101.19;

$$disAPPROX := -30.\ln(.03\cos(.57\,t) + .99\sin(.57\,t)) - 101.19$$

> TmaxAPPROX := solve(velAPPROX = 0, t);

$$TmaxAPPROX := 2.701754386$$

> HmaxAPPROX := subs(t=TmaxAPPROX, disAPPROX);

$$HmaxAPPROX := -30.\ln(.03\cos(1.540000000) + .99\sin(1.540000000)) - 101.19$$

> evalf(HmaxAPPROX);

-100.9022539

END OF WORKSHEET

>

>