```
> restart;
  with( DEtools ): with( plots ):
```

Maple Worksheet with the solution to Problem 36 (p 50)

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The differential equation and initial condition for the first stage in the process are

```
> ode1 := diff( A(t), t ) + 1/100*A(t) = 10;
> ic1 := A(0)=50;
```

This problem can be solved as a linear ODE. The first step is to identify the integrating factor

```
> mu := exp( int( 1/100, t ) );
```

We need to evaluate the integral on the RHS of the equation after multiplying the ODE by the integrating factor

```
> int( 10*mu, t );
```

The equation for the solution now appears as

$$e^{(1/100\,t)}\,A(t) = 1000\,e^{(1/100\,t)} + C$$

Which is easily solved for A. The general solution is

$$A(t) = 1000 + C\,e^{(-1/100\,t)}$$

The initial condition is used to find that C=-950. The amount of salt in the tank, during the first 10 minutes, is

```
> s1 := A(t) = 1000-950*exp(-1/100*t);
```

$$s1 := A(t) = 1000 - 950\,e^{(-1/100\,t)}$$

The differential equation for the second stage of the process (after 10 minutes) is

```
> ode2 := diff( A(t), t ) + 6/(510-t)*A(t) = 10;
```

$$ode2 := \left(\frac{\partial}{\partial t}A(t)\right) + 6\,\frac{A(t)}{510 - t} = 10$$

The initial condition is obtained from the solution to the first stage.

```
> ic2 := subs( t=10, s1 );
```

$$ic2 := A(10) = 1000 - 950\,e^{\left(\frac{-1}{10}\right)}$$

which is approximately

```
> evalf( ic2 );
```

This problem is also a linear ODE, so the procedure for finding the solution is the same as above.

> mu := exp(int(6/(510-t), t));

The integral of the RHS, after mulitiplication by mu, is

> int(10*mu, t);

Thus

$$\frac{\mathrm{A}(t)}{(510 - t)^6} = 2\,\frac{1}{(510 - t)^5} + C$$

from which the general solution is seen to be

> s2 := A(t) = 1020-2*t+C*(510-t)^6;

$$s2 := \mathrm{A}(t) = 1020 - 2\,t + C\,(510 - t)^6$$

All that remains is to solve for the constant C. For this we first substitute the values for t and A(t) into the general solution

> eq := subs({ t=10, A(t)=rhs(ic2) }, s2);

then solve for C

> solve(eq, C);

The solution to the IVP is then found by substituting the above value of C into the general solution.

> s2a := subs(C=", s2);

$$s2a := \mathrm{A}(t) = 1020 - 2\,t - \frac{19}{312500000000000}\,\mathrm{e}^{\left(\frac{-1}{10}\right)}(510 - t)^6$$

> evalf(s2a);

To answer the question posed in the book, we must determine the amount of salt in the tank at t=30 (20 minutes after the leak starts).

> subs(t=30, s2a);

> evalf(");

It’s a shame to have done all this work and end the problem on such an unenlightened note. At the least, the solution should be plotted

> p1 := plot(rhs(s1), t=0..10):

> p2 := plot(rhs(s2a), t=10..30):

> display({ p1, p2 }, title=‘Amount of salt as a function of time‘);

Note that the coefficient of the last term of the solution is very small. At first glance the solution appears to be essentially linear. However, the following plot shows the contribution of this term becomes quite significant as time increases

> p3 := plot(rhs(s2a), t=10..600):

> display({ p1, p3 });

ADDITIONAL QUESTIONS THAT YOU SHOULD CONSIDER

When is the amount of salt in the tank at it’s maximum?

Plot the CONCENTRATION of salt in the tank as a function of time.

Does the concentration ever reach a maximum?

Do you see anything unusual in this plot? (Can the amount of salt be negative?) Explain this behavior.

END OF WORKSHEET

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