```
> restart;
  with( plots ): with( DEtools ):
```

Homework Solutions for Section 1.3

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January 30, 1995

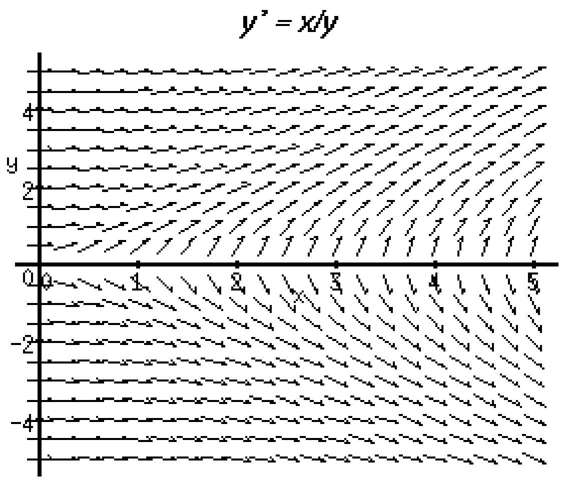
WWW URL: http://www.math.scarolina.edu/~meade/math242/worksheets/soln1-3.ms

Problem # 4 (p. 19)

```
> ODE := diff( y(x), x ) = x/y(x);
```

$$ODE := \frac{\partial}{\partial x}\,\mathrm{y}(x) = \frac{x}{\mathrm{y}(x)}$$

```
> DEplot( ODE, [x,y], x=0..5, y=-5..5, arrows=THIN, title=`y' = x/y` );
```



You can draw the solution through the initial condition (y(0)=-1) by hand. To check that your solution is reasonable, see the plot produced by the following command.

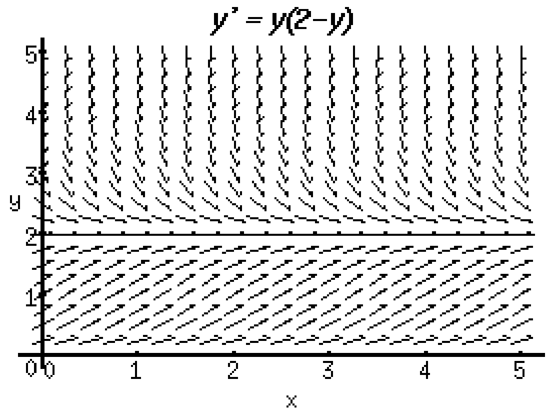
```
> DEplot( ODE, [x,y], x=0..5, { [0,-1] }, y=-5..5, arrows=THIN, title=`y' = x/y` );
```

Problem # 5 (p. 19)

```
> ODE := diff( y(x), x ) = y(x)*(2-y(x));
```

$$ODE := \frac{\partial}{\partial x}\, y(x) = y(x)\,(2 - y(x))$$

```
> DEplot( ODE, [x,y], x=0..5, y=0..5, arrows=THIN, title=‘y’ = y(2-y)‘ );
```



See the following command to check that your

```
> DEplot( ODE, [x,y], x=0..5, { [0,3] }, y=0..5, arrows=THIN, title=‘y’ = y(2-y)‘ );
```

Problem # 10 (p. 19)

```
> restart;
  with( plots ): with( DEtools ):
> ODE := diff( y(x), x ) = f(x,y(x)):
  IC := y(X[0])=Y[0]:
```

Enter the data for the problem here

```
> f := (x,y) -> x/y;
  X[0] := 0;
  Y[0] := -1;
  h := 0.1;
  N := 5;
```

$$f := (x, y) \to \frac{x}{y}$$

$$X_0 := 0$$

$$Y_0 := -1$$

$$h := .1$$

$$N := 5$$

Let's check that the problem has been correctly entered

```
> ODE;
  IC;
```

$$\frac{\partial}{\partial x}\, \mathrm{y}(x) = \frac{x}{\mathrm{y}(x)}$$

$$\mathrm{y}(0) = -1$$

Maple Implementation of Euler's Method
(Do not change the next 8 lines.)

```
> euler_pts := [ X[0],  Y[0] ]:
  for n from 0 to N-1 do
    X[n+1] := X[n] + h;
    Y[n+1] := Y[n] + h*f(X[n],Y[n]);
    new_pt := [ X[n+1], Y[n+1] ];
    euler_pts := euler_pts, new_pt;
    print( new_pt );
  od:
```

$$[.1, -1]$$

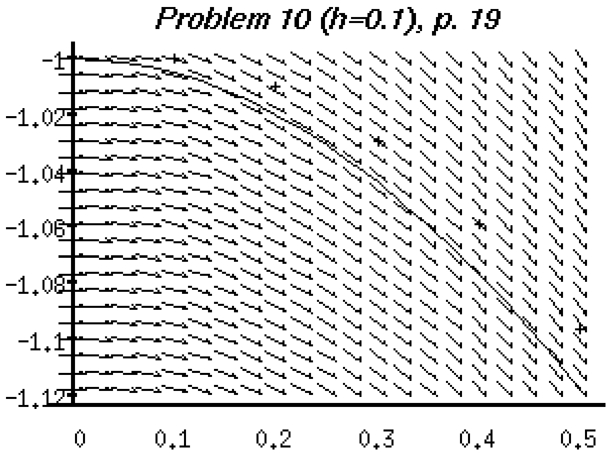
$$[.2, -1.01]$$

$$[.3, -1.029801980]$$

$$[.4, -1.058933794]$$

$$[.5, -1.096707638]$$

```
> euler_plot := plot( [ euler_pts ], style=POINT, color=GREEN ):
  dfield_plot := DEplot( ODE, [x,y], x=0..0.5, { [X[0],Y[0]] }, arrows=THIN ):
  display( {dfield_plot, euler_plot }, title=‘Problem 10 (h=0.1), p. 19‘ );
```



Problem # 11 (p. 19)

```
> f := (x,y) -> y*(2-y);
  X[0] := 0;
  Y[0] := 3;
  h := 0.1;
  N := 5;
```

$$f := (x, y) \rightarrow y\,(2 - y)$$

$$X_0 := 0$$

$$Y_0 := 3$$

$$h := .1$$

$$N := 5$$

Maple Implementation of Euler’s Method
(Do not change the next 8 lines.)

```
> euler_pts := [ X[0],  Y[0] ]:
  for n from 0 to N-1 do
    X[n+1] := X[n] + h;
    Y[n+1] := Y[n] + h*f(X[n],Y[n]);
```

```
    new_pt := [ X[n+1], Y[n+1] ];
    euler_pts := euler_pts, new_pt;
    print( new_pt );
  od:
```

$$[.1, 2.7]$$

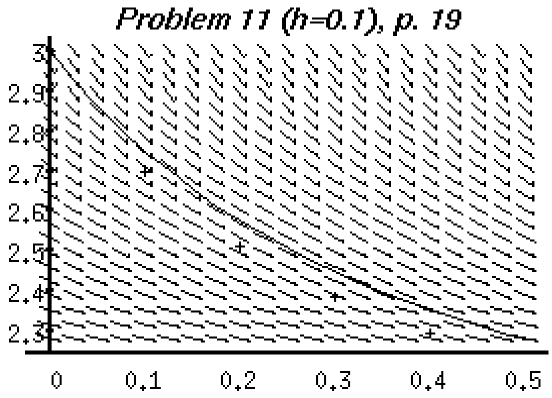
$$[.2, 2.511]$$

$$[.3, 2.3826879]$$

$$[.4, 2.291505317]$$

$$[.5, 2.224706719]$$

```
> euler_plot := plot( [ euler_pts ], style=POINT, color=GREEN ):
  dfield_plot := DEplot( ODE, [x,y], x=0..0.5, { [X[0],Y[0]] }, arrows=THIN ):
  display( {dfield_plot, euler_plot }, title=‘Problem 11 (h=0.1), p. 19‘ );
```



Problem # 22 (p. 20)

A key to this problem is the translation of the variables t and T to x and y. (Remember our discussion in Monday’s class.)

```
> f := (x,y) -> 70-y;
  X[0] := 0;
  Y[0] := 100;
  h := 0.1;
  N := 20;
```

$$f := (x, y) \to 70 - y$$

$$X_0 := 0$$

$$Y_0 := 100$$

$$h := .1$$

$$N := 20$$

Maple Implementation of Euler's Method
(Do not change the next 8 lines.)

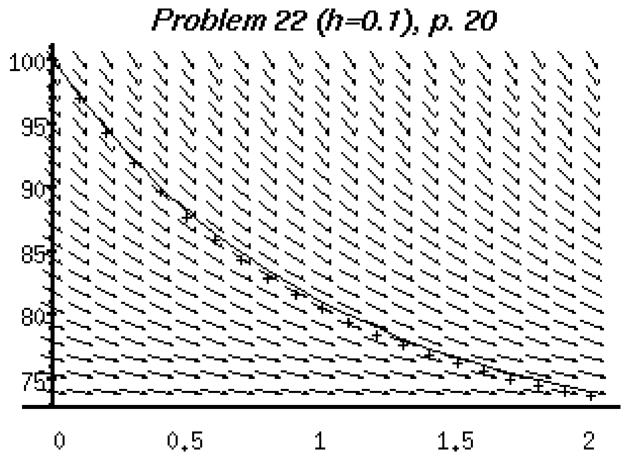
```
> euler_pts := [ X[0],  Y[0] ]:
  for n from 0 to N-1 do
    X[n+1] := X[n] + h;
    Y[n+1] := Y[n] + h*f(X[n],Y[n]);
    new_pt := [ X[n+1], Y[n+1] ];
    euler_pts := euler_pts, new_pt;
    print( new_pt );
  od:
```

$$[1.0, 80.46035320]$$

$$[2.0, 73.64729964]$$

(other output deleted in an attempt to conserve trees and/or electrons)l

```
> euler_plot := plot( [ euler_pts ], style=POINT, color=GREEN ):
  dfield_plot := DEplot( ODE, [x,y], x=0..2, { [X[0],Y[0]] }, arrows=THIN ):
  display( {dfield_plot, euler_plot }, title=‘Problem 22 (h=0.1), p. 20‘ );
```



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