```
> restart;
  with( student ):
  readlib( laplace ):
```

Maple Worksheet to Accompany the Section 9.3

Solving Linear Systems with Laplace Transforms

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File: sec9-3.ms

Note the different packages that are loaded at the start of this worksheet. The student package has a number of useful tools, we will be using completesquare. The readlib should not be necessary, but is suggested to ensure that you are using the most up-to-date version of the Laplace transform procedures..

Example 1 (p. 429 in text)

The solution process of a system of ODEs is the same as for a single equation:

i. apply the Laplace transform to the IVP
ii. solve for the Laplace transform of the solution
iii. apply the inverse Laplace transform

But first, we must enter the initial value problem. First, there are the differential equations

```
> ODE1 := diff( x(t), t ) - 2*y(t) = 4*t;
> ODE2 := diff( y(t), t ) + 2*y(t) - 4*x(t) = -4*t-2;
>
```

then, the initial conditions

```
> IC1 := x(0) = 4;
> IC2 := y(0) = -5;
>
```

STEP 1: Apply Laplace transform to IVP (including use of the initial condition)

```
> lapODE1 := laplace( ODE1, t, s );
> lapODE1 := subs( IC1, lapODE1 );
>
> lapODE2 := laplace( ODE2, t, s );
> lapODE2 := subs( IC2, lapODE2 );
>
```

STEP 2: Solve the system of algebraic equations for the Laplace transform of the functions.

Note that this is a little messy in that there is no simple way to specify the Laplcace transform of the solution.

```
> lapSOL := solve( {lapODE1, lapODE2}, { laplace(y(t),t,s), laplace(x(t),t,s)} );
>
```

STEP 3: Use the inverse Laplace transform to find the solution to the IVP.

Recall that this process typically calls for the use of partial fractions (and sometimes completing the square). These operations occur exactly as before and will result in expressions that appear in the Table of Laplace Transforms.

```
> convert( lapSOL, parfrac, s );
>
```

From this we can SEE that the solutions are

```
  x(t) =  3*exp(-4*t) + exp(2*t)
  y(t) = -6*exp(-4*t) + exp(2*t) - 2*t
```

ADDITIONAL REMARKS:

This entire process can be shortened in a number of places. While I will show you these shortcuts, these do NOT exempt you from understanding what is being done at each stage of the process.

First, Step 3 can be shortened by using Maple’s invlaplace command to apply the inverse Laplace transform.

```
> invlaplace( lapSOL, s, t );
>
```

The absolute shortest way to solve the IVP is to use Maple’s dsolve command (with the Laplace transform option). Here is how that would look:

```
> dsolve( { ODE1, ODE2, IC1, IC2 }, { x(t), y(t) }, method=laplace );
>
```

CONCLUDING REMARKS

Note the form of this solution. As with any other linear ODE, the solution consists of the sum of a solution to the homogeneous equation and a particular solution to the nonhomogeneous equation. Can you identify these parts of the solution?

To gain further insight into the structure of these solutions, repeat the above process without specifying initial conditions. The solution will appear much more complicated, but it’s not really that bad.

> dsolve({ ODE1, ODE2}, { x(t), y(t) }, method=laplace);
>

To see the underlying structure of this solution, collect the exponential terms and look at their coefficients. You should find common factors (involving x(0) and y(0)). You should end up with an expression that looks like

```
  x(t) = -1 * 1/3*(y(0)-x(0)) * exp(-4*t) + 1* 1/3*(y(0)+2*x(0)) * exp(2*t)
  y(t) =  2 * 1/3*(y(0)-x(0)) * exp(-4*t) + 1* 1/3*(y(0)+2*x(0)) * exp(2*t)
- 2*t
```

Further insight into the solution is found by defining

```
  c1 = 1/3*(y(0)-  x(0))
  c2 = 1/3*(y(0)+2*x(0))
```

The solution can now be written as

```
  x(t) = -1 * c1 * exp(-4*t) + 1* c2 * exp(2*t)
  y(t) =  2 * c1 * exp(-4*t) + 1* c2 * exp(2*t) - 2*t
```

Solution to systems of IVPs are also oftern presented in vector notation. In this case this would appear as

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1 \begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{-4t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + \begin{bmatrix} 0 \\ -2t \end{bmatrix}$$

The two linearly independent solutions are now apparent, as is the particular solution The language of linear algebra is also useful here. For example, the vector [-1, 2]’ is an eigenvector with eigenvalue lambda=-4 while the vector [1, 1]’ is an eigenvector with eigenvalue lambda=2. Without going into detail into eigenvalues and eigenvectors, I want to point out that the eigenvalues play the same role as the roots of the characteristic polynomial in the case of a single second order ODE.

END OF WORKSHEET
>