```
> restart;
  with( student ):
  readlib( laplace ):
```

Maple Worksheet to Accompany the Section 7.4

Inverse Laplace Transforms

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File: sec7-4.ms

Note the different packages that are loaded at the start of this worksheet. The student package has a number of useful tools, we will be using completesquare. The readlib should not be necessary, but will overcome the problem that some people encountered with invlaplace.

Now that you have some familiarity with the Laplace transform, it's time to tell you about the Maple commands related to Laplace transforms. Basically, there are two commands: laplace and invlaplace. Each takes three arguments: an expression and two variables. Here are several examples (which we've already seen in class):

Example 1: Find the Laplace transform of the constant function (f=1).

```
> f := 1;
```

$$f := 1$$

```
> F := laplace( f, t, s );
```

$$F := \frac{1}{s}$$

Example 2: Find the Laplace transform of f = t.

```
> f := t;
```

$$f := t$$

```
> F := laplace( f, t, s );
```

$$F := \frac{1}{s^2}$$

*Example 3: Find the Laplace transform of f = 5-exp(2*t) + 6*t^2.*

> f := 5 - exp(2*t) + 6*t^2;

$$f := 5 - e^{(2\,t)} + 6\,t^2$$

> F := laplace(f, t, s);

$$F := 2\,\frac{2\,s^3 - 5\,s^2 + 6\,s - 12}{s^3\,(s - 2)}$$

Note that this answer is written with a common denominator. In order to easily compare this answer with the one found in class, we need to find the PARTIAL FRACTION DECOMPOSITION of F. Here is how that could be done in Maple:

> convert(F, parfrac, s);

$$5\,\frac{1}{s} - \frac{1}{s - 2} + 12\,\frac{1}{s^3}$$

Now it is very simple to check that the solution found in class is consistent with Maple's result.

The Inverse Laplace Transform works similarly.

Example 1 : #1 (p. 300)

> F := 6/(s-1)^4;

$$F := 6 \frac{1}{(s - 1)^4}$$

> f := invlaplace(F, s, t);

$$f := t^3 e^t$$

Example 2 : # 6 (p. 300)

> F := 4/(s^2+9);

$$F := 4 \frac{1}{s^2 + 9}$$

> f := invlaplace(F, s, t);

$$f := \frac{4}{3} \sin(3 t)$$

*Example 3 : Find the Inverse Laplace Transform of F = (2*s+16)/(s^2+4*s+13)*

> F := (2*s+16)/(s^2+4*s+13);

$$F := \frac{2 s + 16}{s^2 + 4 s + 13}$$

> f := invlaplace(F, s, t);

$$f := 4 e^{(-2 t)} \sin(3 t) + 2 e^{(-2 t)} \cos(3 t)$$

Does this answer make sense? Note that the denominator has no REAL VALUED roots, so it's an irreducible quadratic. Thus, partial fractions will not change anything.

> convert(F, parfrac, s);

$$\frac{2 s + 16}{s^2 + 4 s + 13}$$

These situations call for a different approach: COMPLETE THE SQUARE. Once we load the appropriate package, Maple can perform this task for us as well.

> completesquare(F);

$$\frac{2\,s+16}{(s+2)^2+9}$$

*We are still not quite out of the woods. Look at the Table of Laplace Transforms. There are two entries which match denominators of this form. However, the numerators do not quite match with what we have in F. Note, however, that the numerator can be written as 2*s+16 = 2*(s+2)+12. Now, separate the expression into the two separate terms:*

> F2 := 2*(s+2)/((s+2)^2+9) + 12/((s+2)^2+9);

$$F2 := 2\,\frac{s+2}{(s+2)^2+9} + 12\,\frac{1}{(s+2)^2+9}$$

> simplify(F2);

$$2\,\frac{s+8}{s^2+4\,s+13}$$

This should be the same as F. Is it?

> invlaplace(F2, s, t);

$$2\,e^{(-2\,t)}\cos(3\,t) + 4\,e^{(-2\,t)}\sin(3\,t)$$

If this agrees with the earlier response, and agrees with the result obtained using the table, then it must be correct.

The process of finding the Inverse Laplace Transform can be confusing. Here's a quick overview.

1 . Find the partial fraction decomposition of F
2. If there are any irreducible quadratic terms (in the denominator), then complete the square.
3. Now, all terms should match with an entry in the table.

SUGGESTION: Whenever you use the table of Laplace Transforms, list the entry that you use -- including explicit identification of the parameters (a, b, n).

Example 4: #27 (p. 301)

In this problem it is first necessary to do some algebra and solve for F. The result is

> F := 5/(s+1)/(s^2-4);

$$F := 5 \frac{1}{(s+1)(s^2-4)}$$

Let's use the outline given above to use the table to find the inverse Laplace transform of F.

> convert(F, parfrac, s);

$$-\frac{5}{3}\frac{1}{s+1}+\frac{5}{12}\frac{1}{s-2}+\frac{5}{4}\frac{1}{s+2}$$

As this is free of quadratic terms, the inverse Laplace transform of F is seen to be

> f := -5/3*exp(-t) + 5/12*exp(2*t) + 5/4*exp(-2*t);

$$f := -\frac{5}{3}e^{(-t)}+\frac{5}{12}e^{(2t)}+\frac{5}{4}e^{(-2t)}$$

Let's check this

> CHECK := invlaplace(F, s, t);

$$CHECK := -\frac{5}{3}e^{(-t)}+\frac{5}{12}e^{(2t)}+\frac{5}{4}e^{(-2t)}$$

>

END OF WORKSHEET
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