> restart;
with(plots): with(DEtools): with(student):

Maple Worksheet for Sections 5.6

Numerical Methods for Second Order Equations

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File: sec5-6.ms

The text talks about the fourth-order Runge-Kutta method. We have not talked about this method. Regardless, it is simple to rework all the problems with Improved Euler's method.

There are two keys to making this work :

1. Rewrite the second-order IVP as a system of first-order IVPs (thus, identify the ODE and the initial conditions)

2. Replace all occurrences of Runge-Kutta with Improved Euler's in the text and problems.

Almost all numerical methods for higher-order ODEs are based on the fact that the equation can be written as a system of first order ODEs. The extra variable is the derivative of the solution. See the text for a few examples (p. 232)

Once you have the problem re-written as a system, you can use the general code provided in this worksheet to compute approximate solutions to the problems and answer the questions posed in the problems.

1. Right hand sides

```
> f[1] := (t,y1,y2) -> ? ;
  f[2] := (t,y1,y2) -> ? ;
```

2. Initial conditions

```
> t.0    := ? ;
  y1.0 := ? ;
  y2.0 := ? ;
```

3. Stepsize and number of steps

```
> h := ? ;
  N := ? ;
```

Improved Euler's Method for a system of two equations

CHANGE THESE COMMANDS ONLY AT YOUR OWN RISK !!

```
> T := t.0:
  Y1 := y1.0:
  Y2 := y2.0:
  print( [ t.0, y1.0, y2.0 ] );
  for n from 0 to N-1 do
    t.(n+1) := t.n + h;
    F[1] := f[1]( t.n, y1.n, y2.n );
    F[2] := f[2]( t.n, y1.n, y2.n );
    G[1] := f[1]( t.(n+1), y1.n+h*F[1], y2.n+h*F[2] );
    G[2] := f[2]( t.(n+1), y1.n+h*F[1], y2.n+h*F[2] );
    y1.(n+1) := y1.n + h/2*(F[1]+G[1]);
    y2.(n+1) := y2.n + h/2*(F[2]+G[2]);
    T := T, t.(n+1);
    Y1 := Y1, y1.(n+1);
    Y2 := Y2, y2.(n+1);
    print( [t.(n+1), y1.(n+1), y2.(n+1)] );
  od:
```

Basic plots from the computed data

```
> plot( zip( (t,y) -> [t,y], [T], [Y1] ), style=POINT, title='Approximate values for y1 (t vs y1)' );
> plot( zip( (t,y) -> [t,y], [T], [Y2] ), style=POINT, title='Approximate values for y2 (t vs y2)' );
> plot( zip( (y1,y2) -> [y1,y2], [Y1], [Y2] ), style=LINE, title='Phase Plane (y1 vs y2)' );
```

END OF WORKSHEET

```
>
```