>

Maple Worksheet to Accompany Sections 5.2 of Nagel and Saff

Damped Vibrations

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File: sec5-2.ms

In this worksheet I illustrate the use of Maple to find the solution of initial value problems that arise in free motion problems with damping.

Remember that there is more to these problems than solving the differential equation. Once the solution is obtained, you have to use it to provide an answer to a physical problem. For this it is often of use to first have a plot of the solution.

Example 1 (Problem #1, p. 210)

> restart;
> ODE := diff(x(t), t,t) + b*diff(x(t), t) + 16*x(t) = 0;

$$ODE := \left(\frac{\partial^2}{\partial t^2}\,\mathrm{x}(t)\right) + b\left(\frac{\partial}{\partial t}\,\mathrm{x}(t)\right) + 16\,\mathrm{x}(t) = 0$$

> IC1 := x(0)=1;

$$IC1 := \mathrm{x}(0) = 1$$

> IC2 := D(x)(0)=0;

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First, find the solution for b=6:

> ODE6 := subs(b=6, ODE);

$$ODE6 := \left(\frac{\partial^2}{\partial t^2} x(t)\right) + 6\left(\frac{\partial}{\partial t} x(t)\right) + 16\, x(t) = 0$$

> EOM6 := dsolve({ ODE6, IC1, IC2 }, x(t));

$$EOM6 := x(t) = e^{(-3\,t)} \cos\left(\sqrt{7}\ t\right) + \frac{3}{7}\sqrt{7}\ e^{(-3\,t)} \sin\left(\sqrt{7}\ t\right)$$

Then, for b=8:

> ODE8 := subs(b=8, ODE);

$$ODE8 := \left(\frac{\partial^2}{\partial t^2} x(t)\right) + 8\left(\frac{\partial}{\partial t} x(t)\right) + 16\, x(t) = 0$$

> EOM8 := dsolve({ ODE8, IC1, IC2 }, x(t));

$$EOM8 := x(t) = e^{(-4\,t)} + 4\, e^{(-4\,t)}\, t$$

And, lastly, for b=10:

> ODE10 := subs(b=10, ODE);

$$ODE10 := \left(\frac{\partial^2}{\partial t^2} x(t)\right) + 10\left(\frac{\partial}{\partial t} x(t)\right) + 16\, x(t) = 0$$

> EOM10 := dsolve({ ODE10, IC1, IC2 }, x(t));

$$EOM10 := x(t) = -\frac{1}{3} e^{(-8\,t)} + \frac{4}{3} e^{(-2\,t)}$$

Note that in all cases the real part of the roots are negative, hence all solutions are decaying. This occurs because all three coefficients in the ODE are positive. (Do you see why this is so?)

Note that these three solutions are all qualitatively different.

UNDERDAMPED MOTION:
The first solution, with b=6, is an exponentially damped oscillation. This type of motion, which results from complex-valued roots of the characteristic polynomial, is called UNDERDAMPED.

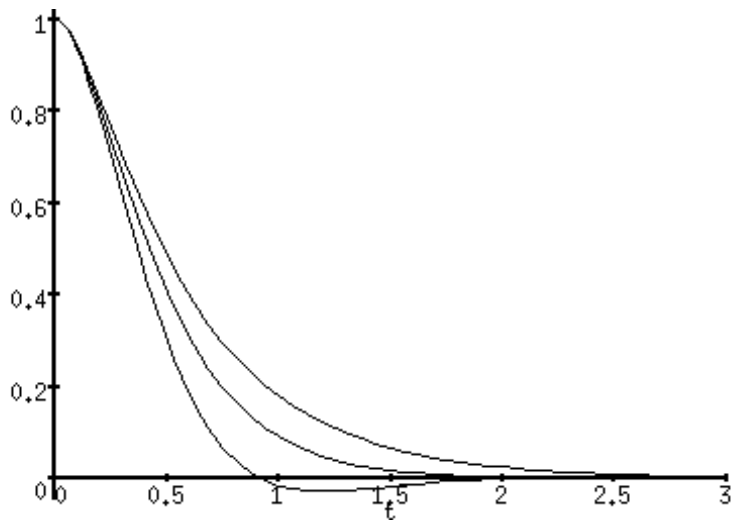
It is usually most useful to write this solution in the form:
   x(t) = exp(alpha*t) * A * sin(beta*t + phi)
The constants alpha and beta are simple to identify; A and phi can be found in the same manner as we have used for undamped motion.

CRITICALLY DAMPED MOTION:
The solution with b=8 is an example of CRITICAL DAMPING. Note that the root of the characteristic equation for this problem is r=-4 (with multiplicity 2).

OVERDAMPED MOTION:
The solution consisting of two decaying exponentials is an example of an OVERDAMPED MOTION. This case is characterized by two distinct real roots.

This problem is complete once the three solutions are plotted. In the interest of space, let’s put all three on the same set of axes.

> plot({ rhs(EOM6), rhs(EOM8), rhs(EOM10) }, t=0..3);



END OF WORKSHEET