```
> 
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Maple Worksheet to Accompany Sections 5.1 of Nagel and Saff

Mechanical Vibrations and Simple Harmonic Motion

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File: sec5-1.ms

Our main use of Maple at this point is to display the solutions of an appropriate IVP. In Wednesday’s class we will also see how to use Maple to find the solution of the IVP.

Example 1 (Problem #1, p. 203)

```
> restart;
> ODE := 100*diff( x(t), t$2 ) + 4900*x(t) = 0;
```

$$ODE := 100\left(\frac{\partial^2}{\partial t^2}\,\mathrm{x}(t)\right) + 4900\,\mathrm{x}(t) = 0$$

```
> IC1 := x(0)=1/10;
```

$$IC1 := \mathrm{x}(0) = \frac{1}{10}$$

```
> IC2 := D(x)(0)=0;
```

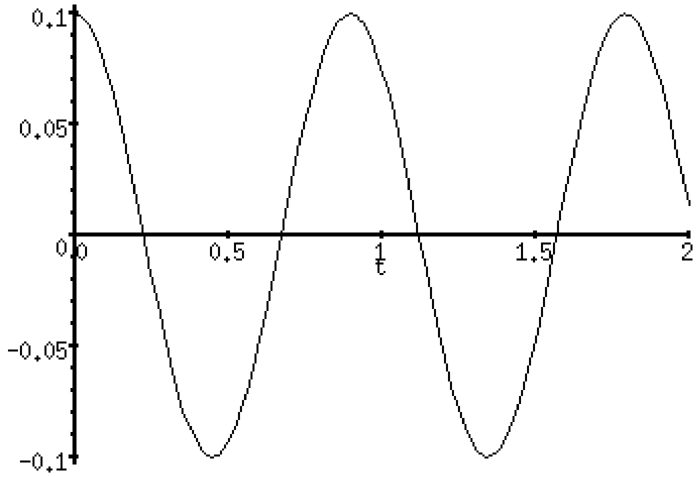
$$IC2 := \mathrm{D}(x)(0) = 0$$

```
> eq_of_motion := dsolve( { ODE, IC1, IC2 }, x(t) );
```

$$eq\_of\_motion := \mathrm{x}(t) = \frac{1}{10}\cos(7\,t)$$

Now that we have this solution, what can we say about it? First, the motion is simple harmonic motion (the amplitude and period are constant). What are the amplitude (A) and period? Approximations can be obtained from the graph

```
> plot( rhs(eq_of_motion), t=0..2 );
```



Exact values for the amplitude and period can be found directly from the equation of motion

```
> A := 1/10;
```

$$A := \frac{1}{10}$$

```
> period := solve( 7*t=2*Pi, t );
```

$$period := \frac{2}{7}\pi$$

Example 2 (modified initial conditions)

The first example was pretty simple. Let’s see what happens if one of the initial conditions is slightly modified, e.g. suppose the system is started by throwing the mass downward with an initial velocity of 1m/sec

```
> restart;
> ODE := 100*diff( x(t), t$2 ) + 4900*x(t) = 0;
```

$$ODE := 100\left(\frac{\partial^2}{\partial t^2}\,\mathrm{x}(t)\right) + 4900\,\mathrm{x}(t) = 0$$

```
> IC1 := x(0)=1/10;
```

$$IC1 := \mathrm{x}(0) = \frac{1}{10}$$

```
> IC2 := D(x)(0)=10;
```

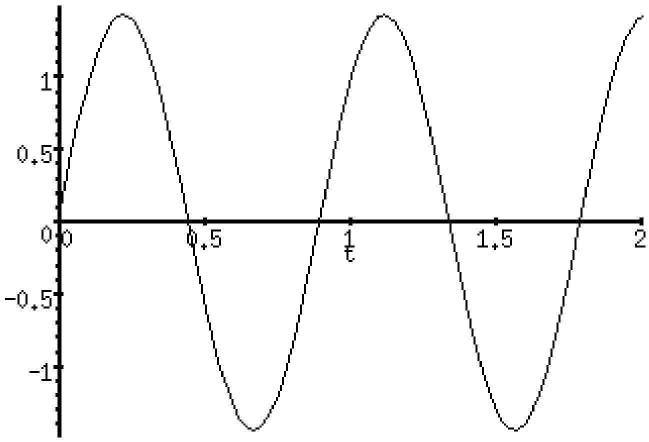
$$IC2 := \mathrm{D}(x)(0) = 10$$

```
> eq_of_motion := dsolve( { ODE, IC1, IC2 }, x(t) );
```

$$eq\_of\_motion := \mathrm{x}(t) = \frac{1}{10}\cos(7\,t) + \frac{10}{7}\sin(7\,t)$$

Notice that the solution is still a simple harmonic motion. But, what are the amplitude and period of this solution? Let’s start with the graph.

```
> plot( rhs(eq_of_motion), t=0..2 );
```



From this plot we see that the amplitude is approximately 0.1733 and the period is approximately 0.90. Note also that the solution would be a pure sine solution (i.e. starts from x=0) if the graph is shifted about 0.9 seconds to the left; the phase shift of this solution is about 0.9.

All the above can be put into an appropriate context if we first write the solution in the form: x(t) = A*sin(omega*t + phi). As done in class, trigonometric identies (sin(a+b)=sin(a)*cos(b)+cos(a)*sin(b)) can be used to show that:

```
> omega := 7;
```

$$\omega := 7$$

A*sin(phi) = 1/10
A*cos(phi) = 1/7

Solving, as demonstrated in class, for A and phi leads to:

```
> A := sqrt( (1/10)^2 + (1/7)^2 );
  evalf( A );
```

$$A := \frac{1}{70}\sqrt{149}$$

$$.1743793661$$

```
> phi := arctan( 1/10, 1/7 );
```

$$\phi := \arctan\left(\frac{7}{10}\right)$$

or, in degrees

```
> evalf( phi*180/Pi );
```

$$34.99202020$$

That is, the solution is a simple harmonic motion with amplitude

```
> A;
```

$$\frac{1}{70}\sqrt{149}$$

period

```
> period := solve( 7*t = 2*Pi, t );
```

$$period := \frac{2}{7}\pi$$

and phase_angle

```
> phi;
```

$$\arctan\left(\frac{7}{10}\right)$$

To see exactly what the role of the phase angle is, let’s plot the solution and the function with phi=0 on the same set of axes

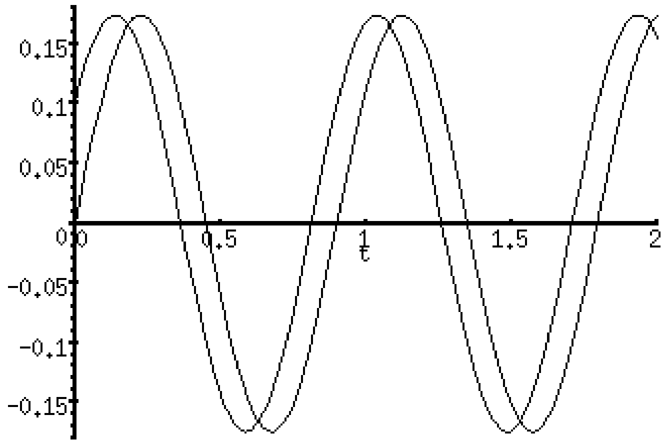
> soln := A*sin(omega*t+phi);

$$soln := \frac{1}{70}\sqrt{149}\,\sin\left(7\,t + \arctan\left(\frac{7}{10}\right)\right)$$

> unshifted_soln := A*sin(omega*t);

$$unshifted\_soln := \frac{1}{70}\sqrt{149}\,\sin(7\,t)$$

> plot({ soln, unshifted_soln }, t=0..2);



Note how the two curves are the same, except for the horizontal translation by -phi/omega (approximately -0.08725 radians).

Final Remarks:

Note that the period is not altered by the change in the initial conditions. (The mass and spring constant are all that influence the period.)

The initial conditions (as well as m and k) have an influence over the other parameters in the solution (amplitude and phase angle).

The book gives formulae for each of these quantities. I strongly advise AGAINST the memorization of these formulae.

I will be modelling the type of understanding that I expect you to have.

END OF WORKSHEET

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