> restart;

Maple Worksheet for Sections 4.8

The Method of Undetermied Coefficients

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Variation of Parameters can be used for any non-homogeneous linear second-order ODE. Note that many times the calculations involved in Variation of Parameters are rather complicated, and then the final solution turns out to be something relatively simple (e.g. a linear polynomial or a single exponential).

If the general form of a particular solution can be identified at the outset, then there is no need to use the Variation of Parameters. The procedure based on this idea is called the Method of Undetermined Coefficients.

There are two critical decisions in the Method of Undetermined Coefficients. First, does the method apply? (i.e., can you guess the form of the general solution?) Second, what is the general form of the particular solution.

EXAMPLE #1 (p. 170)

> # restart;
ODE := diff(y(x), x$2) + 3*diff(y(x), x) + 2*y(x) = 3*x + 1;

EXAMPLE #2 (p. 171)

> # restart;
ODE := diff(y(x), x$2) + 3*diff(y(x), x) + 2*y(x) = exp(3*x);

EXAMPLE #4 (p. 173)

> # restart;
ODE := diff(y(x), x$2) + diff(y(x), x) = 5;

EXAMPLE #5 (p. 173)

> # restart;
ODE := diff(y(x), x$2) - diff(y(x), x) -12*y(x) = exp(4*x);

STEP 1: Find the general solution to the homogeneous equation

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```
> y1 := exp(-x);
> y2 := exp(-2*x);
> hSOLN := c1*y1 + c2*y2;
```

STEP 2: Find any solution to the non-homogeneous equation

Start with your guess for the general form of the particular solution

```
> guess := A*x+B;
```

and substitute it into the differential equation

```
> GOAL := simplify( subs( y(x)=guess, ODE ) );
```

It now becomes a little difficult to find a general set of steps to proceed to the final solution. Fortunately, it's not difficult to do some of these calculations by hand, and ask Maple for assistance when needed. The key is to identify the coefficients of each different term in the above expression. The only way this equation can be satisfied for all values of the independent variable is if the coefficients on the left- and right-hand sides agree. For this example, the equations are simple

```
> EQNS := { 3*A+2*B=1, 2*A=3 };
```

Oftentimes the solution to these equations can be identified without the help of Maple. (This problem is one such example.)

```
> COEFS := solve( EQNS, {A,B} );
> pSOLN := subs( COEFS, guess );
```

This solution agrees with the particular solution found by Variation of Parameters!

STEP 3: Write the general solution to the non-homogeneous equation

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```
> gSOLN := hSOLN + pSOLN;
```

END OF WORKSHEET

```
>
```