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Maple Worksheet for Section 4.7

Superposition and Nonhomogeneous Equations

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We will see that general solutions to non-homogeneous equations can be obtained directly from the general solution to the corresponding homogeneous equation. Before we worry about the details, let's look at a few examples that illustrate the main points.

Let's start with an ODE with a general right-hand side.

> restart;
> ODE := diff(y(x), x,x) - 3*diff(y(x), x) + 2*y(x) = g;

$$ODE := \left(\frac{\partial^2}{\partial x^2}\,y(x)\right) - 3\left(\frac{\partial}{\partial x}\,y(x)\right) + 2\,y(x) = g$$

First, let's look at the general solution of the homogeneous equation:

> g := 0; ODE;

$$g := 0$$

$$\left(\frac{\partial^2}{\partial x^2}\,y(x)\right) - 3\left(\frac{\partial}{\partial x}\,y(x)\right) + 2\,y(x) = 0$$

> dsolve(ODE, y(x));

$$y(x) = _C1\,e^{x} + _C2\,e^{(2x)}$$

Now, let's look at several different non-zero RHSs
> g := x; ODE;

$$g := x$$

$$\left(\frac{\partial^2}{\partial x^2}\,y(x)\right) - 3\left(\frac{\partial}{\partial x}\,y(x)\right) + 2\,y(x) = x$$

> dsolve(ODE, y(x));

$$y(x) = \frac{3}{4} + \frac{1}{2}x + _C1\,e^{x} + _C2\,e^{(2x)}$$

>

> g := x^2; ODE;

$$g := x^2$$

$$\left(\frac{\partial^2}{\partial x^2}\,y(x)\right) - 3\left(\frac{\partial}{\partial x}\,y(x)\right) + 2\,y(x) = x^2$$

> dsolve(ODE, y(x));

$$y(x) = \frac{7}{4} + \frac{3}{2}x + \frac{1}{2}x^2 + _C1\ e^{x} + _C2\ e^{(2x)}$$

>
> g := exp(2*x);

$$g := e^{(2x)}$$

> dsolve(ODE, y(x));

$$y(x) = -e^{(2x)} + x\,e^{(2x)} + _C1\ e^{x} + _C2\ e^{(2x)}$$

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What patterns do you notice in the FORM of these solutions?

This single example does not prove anything. Gather more confidence in your observations by re-working the above examples for an ODE of your choice. (Execute restart;, then change the coefficients of ODE and have Maple solve the homogeneous equation and the three non-homogeneous equations.)

Be sure to note any irregularities that you encounter.

Also, feel free to try different expressions for the RHS of the ODE.

SUMMARY:

The general solution to a nonhomogeneous linear ODE always consists of the sum of
i) the general solution to the corresponding homogenous ODE and
ii) a particular solution to the nonhomogeneous ODE.

We know how to find the general solution to any homogeneous ODE. We do not yet know how to find a particular solution to a nonhomogeneous ODE. We'll learn two methods: Variation of Parameters (4.10) and The Method of Undetermined Coefficients (4.8).

END OF WORKSHEET
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