> restart:

Maple Worksheet to Accompany Section 4.6

Characteristic Polynomials with Complex Roots

Prepared by Douglas Meade
15 March 1995

Let's consider the IVP: y'' + y = 0, y(0)=1, y'(0)=2

The characteristic polynomial is : r^2 + 1 = 0 with roots r = I and r = -I.
This gives the appearance that the solutions might be complex valued. Let's see what really happens.

The general solution to the ODE can be written as

> y := c1 * exp(I*x) + c2 * exp(-I*x);

$$y := c1\ e^{(I\,x)} + c2\ e^{(-I\,x)}$$

The constants c1 and c2 will be determined from the initial conditions

> e1 := subs(x=0, y) = 1;

$$e1 := c1\ e^{0} + c2\ e^{0} = 1$$

> e2 := subs(x=0, diff(y,x)) = 2;

$$e2 := I\ c1\ e^{0} - I\ c2\ e^{0} = 2$$

> C := solve({e1,e2}, {c1,c2});

$$C := \left\{ c2 = \frac{1}{2} + I,\ c1 = \frac{1}{2} - I \right\}$$

NOTE that these constancts are complex-valued. When inserted into the general solution we will have a real* mess.

> yy := subs(C, y);

$$yy := \left(\frac{1}{2} - I \right) e^{(I\,x)} + \left(\frac{1}{2} + I \right) e^{(-I\,x)}$$

This is a difficult expression to understand. It would be much simpler if we could explicitly identify the real and imaginary parts of the solution. Maple's evalc command is what we need.

> evalc(yy);

$$\cos(x) + 2\sin(x)$$

This is a real-valued solution. What happened to the imaginary part?

The above example is not a coincidence. Change the initial conditions to any values you choose, then solve for c1 and c2 and simplify the solution to the IVP. THE RESULT WILL ALWAYS BE REAL-VALUED!

* no pun intended

The fact that the solution is real-valued suggests that we might be able to completely avoid working with complex-valued solutions. The key will be to identify two linearly independent real-valued solutions to the ODE. The general answer is to use the real and imaginary parts of one of the complex-valued solutions

```
> y1 := evalc( Re( exp( I*x ) ) );
```

$$y1 := \cos(x)$$

```
> y2 := evalc( Im( exp( I*x ) ) );
```

$$y2 := \sin(x)$$

Thus the general solution can be written as

```
> Y := c1*cos(x) + c2*sin(x);
```

$$Y := c1 \cos(x) + c2 \sin(x)$$

The solution to the IVP is found in the same way as before.

```
> E1 := subs( x=0, Y ) = 1;
```

$$E1 := c1 \cos(0) + c2 \sin(0) = 1$$

```
> E2 := subs( x=0, diff(Y,x) ) = 2;]
```

$$E2 := -c1 \sin(0) + c2 \cos(0) = 2$$

```
> C := solve( {E1,E2}, {c1,c2} );
```

$$C := \{ c1 = 1, c2 = 2 \}$$

```
> YY := subs( C, Y );
```

$$YY := \cos(x) + 2 \sin(x)$$

Note that this is exactly the same solution obtained earlier.

```
>
```

END OF WORKSHEET