```
> restart;
  with( plots ): with( DEtools ): with( student ):
```

Maple Worksheet for Sections 4.10

Variation of Parameters

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File: sec4-10.ms

NOTE: It is perfectly accepatable for you to use this worksheet to assist with the homework problems. The only condition is that you include all the major steps (homogeneous solution, form of particular solution, system of equations, dv1 and dv2, v1 and v2, the particular solution) in your solution. It is NOT ACCEPTABLE to simply state the final solution.

Variation of Parameters is a technique that can be used to find the particular solution for ANY non-homogeneous second-order linear ODE in NORMAL FORM. The first step is to recall that the general solution to the homogeneous equation is

```
> hSOLN := c1*y1 + c2*y2:
```

where y1 and y2 are linearly independent solutions to the corresponding homogeneous equation and c1 and c2 are constants.

Recall our discovery last Friday that the particular solution appears in the same form, but with the constants c1 and c2 replaced by functions v1 and v2:

```
> pSOLN := v1*y1 + v2*y2;
```

We have just seen that the derivatives of v1 and v2, denoted dv1 and dv2, satisfy a system of two linear equations

```
> VPeqns := {        y1 * dv1 +        y2 * dv2 = 0,
             Diff(y1,x) * dv1 + Diff(y2,x) * dv2 = g };
```

The particular solution is found by solving these equations, integrating the derivatives to find v1 and v2, then substituting the final results into the general form of the particular solution. The general solution for the non-homogeneous equation will then be

```
> gSOLN := hSOLN + pSOLN;
```

NOTE: While this process is not complicated, the mechanical operations can be prohibitive, as we will now see.

Select one of the following examples (or enter your own).

EXAMPLE #1 (p. 170)

```
> # restart;
  ODE := diff( y(x), x$2 ) + 3*diff( y(x), x ) + 2*y(x) = 3*x + 1;
```

EXAMPLE #2 (p. 171)

```
> #  restart;
  ODE := diff( y(x), x$2 ) + 3*diff( y(x), x ) + 2*y(x) = exp(3*x);
```

EXAMPLE #3 (p. 171)

```
> # restart;
  ODE := diff( y(x), x$2 ) - diff( y(x), x ) - y(x) = sin(x);
```

EXAMPLE #4 (p. 172)

```
> # restart;
  ODE := diff( y(x), x$2 ) - diff( y(x), x ) = 5;
```

EXAMPLE #5 (p. 173)

```
> # restart;
  ODE := diff( y(x), x$2 ) - diff( y(x), x ) - 12*y(x) = exp(4*x);
```

EXAMPLE #1 (p. 187)

```
> # restart;
  ODE := diff( y(x), x$2 ) + y(x) = tan(x);
```

STEP 1: Find linearly independent solutions to the homogeneous equation

```
> BASIS := dsolve( lhs(ODE)=0, y(x), output=basis );
> y1 := BASIS[1];
> y2 := BASIS[2];
```

STEP 1a: Identify the RHS of the non-homogenouse equation

```
> g := rhs( ODE );
```

STEP 1c: Identify the three different solutions that we have found or will find.

```
> hSOLN := c1*y1 + c2*y2 ;
> pSOLN := v1*y1 + v2*y2 ;
> gSOLN := hSOLN + pSOLN ;
```

STEP 2: Setup and solve the equations for the derivatives of v1 and v2.

```
> VPeqns := {          y1 * dv1 +          y2 * dv2 = 0,
                diff(y1,x) * dv1 + diff(y2,x) * dv2 = g };
> solve( VPeqns, { dv1, dv2 } );
> assign( " );
```

STEP 3: Integrate to find v1 and v2; substitute these into the particular solution and simplify.

```
> v1 := int( dv1, x );
> v2 := int( dv2, x );
> pSOLN;
> simplify( " );
> simplify( gSOLN );
```

END OF WORKSHEET