```
> restart;
  with( DEtools ):
```

Maple Worksheet for Section 2.1 - Motion of a Falling Body

Math 242 (Section 1)
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WWW URL: http://www.math.scarolina.edu/~meade/math242/worksheets/sec2-1.ms

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The differential equation for the velocity of a falling object being acted upon by gravity and air resistance is

```
> ode := m*diff( v(t), t ) = m*g-k*v(t);
```

$$ode := m\left(\frac{\partial}{\partial t}\,\mathrm{v}(t)\right) = m\,g - k\,\mathrm{v}(t)$$

Let’s begin by looking at a few solution curves for a specific set of constants.

```
> m:=1;
  g:=9.8;
  k:=0.2;
```

$$m := 1$$

$$g := 9.8$$

$$k := .2$$

```
> DEplot( ode, [t,v], t=0..10, v=-10..100, arrows=THIN, title=‘Direction Field‘ );
> DEplot( ode, [t,v], t=0..20, { [0,-10], [0,0], [0,25], [0,50], [0,75] }, title=‘Solution Curves‘ );
```

Maple is unable to work with floating point numbers when solving differential equations. We have two options: i) express the constants as rational numbers (e.g. g := 98/10; k := 2/10;) and ii) solve the equation without specifying the constants.

Since it is more general, let's pursue the latter of these. The following commands reset m, g, and k to unevaluated names.
(It is also possible to achieve the same result by re-executing the restart command and definition of ode.)

```
> m := 'm';
  g := 'g';
  k := 'k';
```

$$m := m$$

$$g := g$$

$$k := k$$

```
> dsolve( ode, v(t) );
```

$$v(t) = \frac{m\,g + e^{\left(-\frac{k\,t}{m}\right)}\,_C1\,k}{k}$$

To investigate the behavior of these solutions for large values of t, we need to look at the limit as t -> infinity.

```
> limit( rhs(''), t=infinity );
```

$$\lim_{t \to \infty} \frac{m\,g + e^{\left(-\frac{k\,t}{m}\right)}\,_C1\,k}{k}$$

Maple is unable to evaluate this limit because it knows nothing about the constants k and m. However, we know that these are positive constants and thus the exponential must decay to zero as t increases to infinity. The limiting velocity can be seen to be m*g/k; this is also known as the terminal velocity.

END OF WORKSHEET

```
>
```