> restart:
with(plots): with(DEtools):

Homework # 2 --- Solutions

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#1. Finding derivatives with Maple is just like plotting: plot(expression, variable); and diff(expression, variable);

a. Note that I do not say x(t) :=
GENERAL RULE: do not use parentheses ( ) on the right side of an assignment

> x := 3*exp(-4*t)+t*exp(-4*t)+tan(t^2);

$$x := 3\,e^{(-4\,t)} + t\,e^{(-4\,t)} + \tan(t^2)$$

> diff(x, t);

$$-11\,e^{(-4\,t)} - 4\,t\,e^{(-4\,t)} + 2\left(1 + \tan(t^2)^2\right) t$$

b. The exact same ideas are used for implicit differentiation. Recall that it is necessary, when working this problem by hand, to think of y as a function of x, i.e. y(x). Well, Maple can't (and won't) make this assumption for us. We have to be explicit about this when we enter the function.

But first, I have to erase the value given to x in the first part of the problem.

> x := 'x';

$$x := x$$

> EQN := exp(x*y(x))+y(x)=x-1;

$$EQN := e^{(x\,y(x))} + y(x) = x - 1$$

Note the use of both := and = in this command. The equation EQN is now an equation (not simply an expression). Maple will see this when we ask for the derivative.

> dEQN := diff(EQN, x);

$$dEQN := \left(y(x) + x\left(\frac{\partial}{\partial x} y(x)\right)\right) e^{(x\,y(x))} + \left(\frac{\partial}{\partial x} y(x)\right) = 1$$

You should now be able to enter a simple command that solves for the derivative, dy/dx, in terms of both x and y.

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 # 2. Use the properties of exponentials and logarithms to solve for y.

> EQN := exp(3*y^2) = 2*exp(x*y-ln(x^2));

$$EQN := e^{(3\,y^2)} = 2\,e^{(x\,y\,-\,\ln(x^2))}$$

It might seem as though this is a very simple problem -- just use solve

> solve(EQN, y);

The problem here is that Maple is too smart for our needs. It is possible to instruct Maple to think of x and y as real-valued variables, and to assume x<>0, but this is much more difficult than we are ready to handle. It's much simpler to do these calculations by hand.

 # 3. Global max and min

> x := 5/3*exp(-2*t)*sin(6*t);

$$x := \frac{5}{3}\,e^{(-2\,t)}\,\sin(6\,t)$$

The first step should be to plot the function. The plot will give us a general idea about the shape of the function, including where the global max and min might occur.

> plot(x, t=0..10);

What are your approximate locations for the global max and min? How many critical points does this function have?

To find the exact locations, we use calculus to find the critical points, i.e. solve the equation x'(t)=0.

> dx := diff(x, t);

$$dx := -\frac{10}{3}\,e^{(-2\,t)}\,\sin(6\,t) + 10\,e^{(-2\,t)}\,\cos(6\,t)$$

> CritPt := solve(dx = 0, t);

$$CritPt := \frac{1}{6}\arctan(3)$$

What is the approximate numerical value for this critical point

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*Maple seems to be telling us that there is just this one critical point. But, recall that there are many angles theta with theta=1/6*arctan(3), i.e. tan(6*theta)=3. We can use Maple's plot command to see exactly what I mean!*

> plot({tan(6*t), 3}, t=0..2, -10..10, discont=true);

Look at the crossings of the two curves. The first occurs at approximately 0.21 --not too far from the value found earlier. The other crossings are also critical points. From the original picture, we can see that the second critical point (theta ~= 0.735) is the global min.

There is another approach that could also be used here. Notice that the exponential can be FACTORed from the deriviative.

> factor(dx);

$$-\frac{10}{3}\,e^{(-2\,t)}\,(\sin(6\,t)-3\cos(6\,t))$$

Then, the quantity in parentheses is like what we saw on the Algebra Review. Proceeding as in those examples, the derivative can be written as

> dx_new := -10/3*exp(-2*t)*sqrt(10)*sin(6*t+arctan(-3));

$$dx_new := -\frac{10}{3}\,e^{(-2\,t)}\,\sqrt{10}\,\sin(6\,t-\arctan(3))$$

Recall that the sine function is zero whenever the argument is an integer multiple of Pi.

The first critical point should be

> CP1 := solve(6*t-arctan(3)=0*Pi, t);

$$CP1 := \frac{1}{6}\arctan(3)$$

> evalf(CP1);

$$.2081742954$$

which agrees with our earlier results. The second critical point occurs at

> CP2 := solve(6*t-arctan(3)=1*Pi, t);

$$CP2 := \frac{1}{6}\arctan(3)+\frac{1}{6}\,\pi$$

> evalf(CP2);

$$.7317730712$$

which is pretty close to what we saw graphically.

How would you find the value of the function at the global max and global min?

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>
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>

 # 4. Integrals are similar to derivatives (which were similar to plots). The integrate command is int.

> x := 'x';

$$x := x$$

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a. This is a pretty simple indefinite integral.

> int(2*x^(-1), x);

$$2\ln(x)$$

Note that Maple does not include the + C. You need to add it yourself.

*b. This integral needs to computed using the substitution (u=a*x^2). Maple does all this for us!*

> int(x*exp(-a*x^2), x);

$$-\frac{1}{2}\frac{e^{(-a\,x^2)}}{a}$$

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c. To work this problem by hand it is necessary to first find the partial fraction factorization of the integrand. (We did this in the previous HW!) But, if all we need is the integral, Maple can do this with automatically.

Note the change in the second argument for a definite integral.

> int((2*s^2+3*s+3)/(s*(s+1)^2), s=1..3);

$$3\ln(3) - \frac{1}{2} - \ln(4) + \ln(2)$$

 # 5. This improper integral requires integration by parts. It is simplest (when working the problem by hand) to find the indefinite integral, then use limits to evaluate the improper integral.

In Maple we can work directly with the improper integral

> int(t*exp(-s*t), t=0..infinity);

$$\lim_{t \to \infty-} \frac{-s\,t\,e^{(-s\,t)} - e^{(-s\,t)}}{s^2} + \frac{1}{s^2}$$

Why is Maple unable (or unwilling) to evaluate this limit? (Where do you need the information that s>0?)

The final step (evaluating the limit) is best done by hand; the final answer will be 1/s^2.

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