```
> restart:
  with( plots ): with( DEtools ):
```

Maple Solutions to Homework #1 -- Algebra Review

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#1. a

```
> solve( x^2+4*x+3, x );
```

$$-1, -3$$

#1. b

```
> solve( r^2+5*r+1, r);
```

$$-\frac{5}{2}+\frac{1}{2}\sqrt{21}, -\frac{5}{2}-\frac{1}{2}\sqrt{21}$$

#1. c

```
> solve( s^2+s+1, s );
```

$$-\frac{1}{2}+\frac{1}{2}I\sqrt{3}, -\frac{1}{2}-\frac{1}{2}I\sqrt{3}$$

#2. For this problem, let's start be entering the three equations (note the different uses of := and =)

```
> eq1 := c1+c2-c3=2;
```

$$eq1 := c1 + c2 - c3 = 2$$

```
> eq2 := -c1+2*c2+c3=1;
```

$$eq2 := -c1 + 2\,c2 + c3 = 1$$

```
> eq3 := 2*c1+c2+c3=0;
```

$$eq3 := 2\,c1 + c2 + c3 = 0$$

Now, we again want to SOLVE some equations. The only difference is that there are now three equations; these must be specified as a set. The three unknowns are also specified as a set.

```
> SOLN := solve( {eq1,eq2,eq3}, {c1,c2,c3} );
```

$$SOLN := \{ c1 = 0, c3 = -1, c2 = 1 \}$$

We should all get in the habit of checking our answers. In this problem this amounts to SUBStituting the solution into each of the three equations.

> subs(SOLN, eq1);

$$2 = 2$$

This looks good.

Now, you enter the appropriate commands to check the remaining two equations.

>

>

#3. As far as Maple is concerned, this is no different than the previous problem. (It's really no different to use; it's less pleasant for most of us only because their are functions involved.)

> eq1 := v1(x)*cos(x)+v2(x)*sin(x)=0;

$$eq1 := \mathrm{v1}(x)\cos(x) + \mathrm{v2}(x)\sin(x) = 0$$

> eq2 := -v1(x)*sin(x)+v2(x)*cos(x)=tan(x);

$$eq2 := -\mathrm{v1}(x)\sin(x) + \mathrm{v2}(x)\cos(x) = \tan(x)$$

What command will you use to find the solution to these equations?

> SOLN := solve({eq1,eq2}, {v1(x),v2(x)});

$$SOLN := \left\{ \mathrm{v1}(x) = -\frac{\tan(x)\sin(x)}{\sin(x)^2 + \cos(x)^2}, \mathrm{v2}(x) = \frac{\cos(x)\tan(x)}{\sin(x)^2 + \cos(x)^2} \right\}$$

> simplify(SOLN);

$$\left\{ \mathrm{v1}(x) = \frac{\cos(x)^2 - 1}{\cos(x)}, \mathrm{v2}(x) = \sin(x) \right\}$$

4. This is the first problem that requires a significant amount of thought. Once you have the right approach, the mechanical details are pretty simple. The key is to use the addition formula for sine: sin(u+v)=sin(u)*cos(v) + cos(u)*sin(v).

a.

> ORIG := cos(3*t) + 4/3*sin(3*t);

$$ORIG := \cos(3\,t) + \frac{4}{3}\sin(3\,t)$$

The goal is to find A, omega, and theta so that the expression can be written as

> omega := 'omega': A := 'A': theta := 'theta': # reset all variables

> ANS := A*sin(omega*t+theta);

$$ANS := A \sin(\omega t + \theta)$$

After a little work, this problem boils down to:

> omega := 3;

$$\omega := 3$$

> eq1 := A*sin(theta) = 1;

$$eq1 := A \sin(\theta) = 1$$

> eq2 := A*cos(theta) = 4/3;

$$eq2 := A \cos(\theta) = \frac{4}{3}$$

Note that these equations are exactly the ones that you would encounter when converting the point (4/3, 1) to polar coordinates. As such:

> A := sqrt(1 + (4/3)^2);

$$A := \frac{5}{3}$$

> theta := arctan(1 / (4/3));

$$\theta := \arctan\left(\frac{3}{4}\right)$$

This angle is the exact answer. To see an approximate numerical value of the angle, it is necessary to have the expression EVALuated as a Floating point number.

> evalf(");

$$.6435011088$$

The conversion to degrees is equally simple:

> evalf(theta*180/Pi);

$$36.86989765$$

The final answer would be (note how the most recent values of A and theta are substituted into this answer)

> ANS;

$$\frac{5}{3} \sin\left(3\,t + \arctan\left(\frac{3}{4}\right)\right)$$

which can be plotted (together with the orginal expression). What do you expect to see?

> plot({ORIG,ANS}, t=0..2*Pi);

b. In this case the numbers are a little messier, but the process is the same.

> ORIG := 1/sqrt(2)*cos(2*sqrt(3)*t) - 1/3*sin(2*sqrt(3)*t);

$$ORIG := \frac{1}{2}\sqrt{2}\,\cos\left(2\sqrt{3}\,t\right) - \frac{1}{3}\sin\left(2\sqrt{3}\,t\right)$$

The goal is to find A, omega, and theta so that the expression can be written as

---

> omega := 'omega': A := 'A': theta := 'theta': # reset all variables

---

> ANS := A*sin(omega*t+theta);

$$ANS := A\,\sin(\omega\,t + \theta)$$

After a little work, this problem boils down to:

> omega := 2*sqrt(3);

$$\omega := 2\sqrt{3}$$

> eq1 := A*sin(theta) = 1/sqrt(2);

$$eq1 := A\,\sin(\theta) = \frac{1}{2}\sqrt{2}$$

> eq2 := A*cos(theta) = -1/3;

$$eq2 := A\,\cos(\theta) = \frac{-1}{3}$$

We could use the exact same procedure as above, but let's see if we can make more intelligent use of Maple. I want to solve the above equations for A and theta. Why don't we just use solve?

> solve({eq1,eq2}, {A,theta});

$$\left\{ A = \frac{1}{4}\frac{\left(1 + \mathrm{RootOf}\left(3\,_Z^2 - 2\sqrt{2}\,_Z - 3\right)^2\right)\sqrt{2}}{\mathrm{RootOf}\left(3\,_Z^2 - 2\sqrt{2}\,_Z - 3\right)},\ \theta = 2\,\arctan\left(\mathrm{RootOf}\left(3\,_Z^2 - 2\sqrt{2}\,_Z - 3\right)\right) \right\}$$

Well, maybe there is a reason not to use solve. But, maybe this isn't so bad, let's ask for this answer in floating point arithmetic

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> SOLN := evalf(");

$$SOLN := \{A = -.7817359598,\ \theta = -1.130285664\}$$

This is more like what we expect to see. To check that this is actually correct, we still need to ASSIGN this result to the variables A and theta. (To see what I mean, check Maple's current value for A and theta. This is the distinction between := and =)

> A;

$$A$$

> theta;

$$\theta$$

> assign(SOLN);

> A;

$$-.7817359598$$

> theta;

$$-1.130285664$$

What is the angle, in degrees?

>

Why don't you create a plot that shows both forms of the function in one window.

>

#5. Partial fractions. I have already done the full calculations by hand, there is no reason to repeat that process in Maple. The best way to accomplish this in maple is to make a request to CONVERT an expression to its PARtial FRACtion form.

NOTE: It is necessary to include the variable in which the expansion is performed.

> f := (x^2-26*x-47)/(x-1)/(x+2)/(x+5);

$$f := \frac{x^2 - 26\,x - 47}{(x - 1)\,(x + 2)\,(x + 5)}$$

> convert(f, parfrac, x);

$$-4\,\frac{1}{x - 1} - \frac{1}{x + 2} + 6\,\frac{1}{x + 5}$$

One way to check this answer (besides comparing with our own work) is to ask Maple to SIMPLIFY the partial fraction. The result should be the expression, with a common denominator.

> simplify(");

$$\frac{x^2 - 26\,x - 47}{(x - 1)\,(x + 2)\,(x + 5)}$$

b. The exact same steps can be used for this problem.

> Y := (2*s^2+3*s+3)/(s*(s+1)^2);

$$Y := \frac{2\,s^2 + 3\,s + 3}{s\,(s+1)^2}$$

> convert(Y, parfrac, s);

$$3\,\frac{1}{s} - 2\,\frac{1}{(s+1)^2} - \frac{1}{s+1}$$

> simplify(");

$$\frac{2\,s^2 + 3\,s + 3}{s\,(s+1)^2}$$

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c. OK, now you enter the commands for the third part.

>

>

>

END OF WORKSHEET