>

INTRODUCTION TO MAPLE:
Basic computations and plotting

Math 242 section 001 -- Spring 1995

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14 January 1995

Filename: day1.ms

Objectives:

- Enter mathematical expressions in Maple
- Plot a given function in Maple
- Extract information from a graph

Maple commands used in this worksheet

```
:=
;
:
+, -, *, /, ^
sin, cos, tan
plot
E, Pi
..
{}
help, ?
```

WHAT IS MAPLE?

Much of your mathematical background has probably be focused on developing the ability to solve equations and explore functions. The sophisticated manipulation of symbols and expressions that you use to solve equations and investigate functions can also be performed by software packages called computer algebra systems (CAS). A CAS can be use to generate the exact symbolic solutions you obtained by hand, the numerical approximations you found using a calculator, and the graphs you have drawn. Maple is one of several CASs; other major examples are Mathematica, Macsyma, Derive, and MathCad.

HOW WILL MAPLE BE USED IN THIS COURSE?

Our first, and largest, use of Maple will be the graphical display of functions, including the solution of one or more differential equations. Two other uses will be as a symbolic calculator and as a programming environment for numerical methods for solving differential equations.

Much of this week will be spent introducing Maple and learning how it can be used as a mathematics tool. I expect each of you to be able to use Maple effectively. We will (hopefully) use Maple on a daily basis in this course. You will be expected to use Maple on all homework, projects, and exams.

Remember the analogy that a CAS is to calculus as a calculator is to arithmetic.

Note: In previous semesters, students have reported that Maple has been useful in other engineering and science courses!

DOCUMENTATION and ON-LINE HELP

The on-line help for Maple is very good -- once you learn how to use it. The help pages describe the syntax of each command, a brief description of the algorithm that has been implemented, and a few examples illustrating the use of the command. The examples can be very helpful. I will often cut and paste an example from a help page into my worksheet, confirm that the example works as advertised, then modify the arguments to answer my question.

To obtain help on the command func, type: ?func.

Of course, if you don't know the name of the command, it is a different approach must be used. I find the Keyword Search (located under the Help menu) particularly helpful.

WHERE CAN I USE MAPLE?

Maple is available on many PCs, Macs, and workstations across the USC campus, including SWGN 1D11, 1D15, 1D29, 300Main B203, PSC 106, LC 310, 311, 412. For more specific information, please consult the local support staff.

You may purchase the Student Edition of Maple from the Russell House Bookstore for approximately $90. If you don't see Maple on the shelves, additional copies can be ordered from the Book Information desk. (The full Maple costs more than $400. The difference is only in the size of the objects that can be manipulated. The limits in the Student Edition are more than enough for our needs!)

GETTING STARTED

To run Maple, open the Maple V, Release 3 folder then double-click on the icon labelled Maple V R 3 for Windows. (You will almost always want to Maximize the window.)

To open a workseet, select Open from the File menu, then locate the file you want to load. (You can also click on the icon that looks like an opening file folder located along the top of the Maple window. The icon for New is the blank page at the far left.)

Once in a worksheet the current location is marked by a vertical bar. You can move around by moving the mouse and then clicking the left button once, by using the arrow keys, or by using the up and down arrows on the scroll bar located along the right side of this window followed by finer adjustment with the mouse. A command is entered and executed by positioning the cursor anywhere on the desired line and pressing the return key.

It is a good idea to begin EVERY worksheet that you create with the following two commands. Information about these commands can be found in the on-line help: ?plots, ?restart

```
> restart:
> with( DEtools ): with(plots):
>
```

BASIC FACTS

Maple is a computer language; it cannot read your mind. Maple does follow your instructions to the letter. This means that you need to learn to communicate your needs to Maple in a way that it understands.

Some basic symbols and other fundamentals:

- assignments are made with := (plain = has a different meaning)
  - -- this gives the value of the right hand side (RHS) to the name on the LHS

- every command is terminated by a semi-colon (;) or colon (:)
  - -- with the latter the computation is done, but the result is not displayed

- the double quote (") refers to the result of the immediately preceding computation

- Maple is case sensitive
  - -- that is, the names x and X are different, pi and Pi are not the same thing

- { }
  - -- set notation

- .. (as in a .. b)
  - -- this is how Maple indicates the interval [a, b], that is, the real numbers from a to b

- ? or help
  - -- a request to Maple for information

The standard mathematical functions are denoted by their standard names:
+ (plus) , - (minus) , * (times) , / (divided by) , ^ (raised to the power) , sin, cos, tan, abs, sqrt

Common constants are predefined in Maple, their names are: Pi, E, and I
(Yes, they have to be capitalized exactly like this; PI, pi, e, and i do not refer to these constants.)

In the following lines, use paper and pencil to first predict what you think Maple will do. Then execute the command and see what actually happens!

```
>
> number := 4 *  6 + 12 /  6 - 1 ;
```

$$number := 25$$

```
> power := (-3)^3 ;
```

$$power := -27$$

```
> abs( " );
```

$$27$$

```
> Pi ;
```

$$\pi$$

```
> v:= sin( Pi / 4 ) :
> w:= v^2  ;
```

$$w := \frac{1}{2}$$

```
> v;
```

$$\frac{1}{2}\sqrt{2}$$

```
> tan(  -Pi / 2 ) ;
Error, (in tan) singularity encountered

> 3 / ( 5 - sqrt( number ) ) ;
Error, division by zero

>
```

Now enter some commands of your own! To produce the input prompt either select Insert New Region (Below) from the Format menu, click on the eighth icon from the left, or simply press CTRL-I. Any of these will produce a new input region below the current cursor position. Be generous with the space bar to make your commands easy to read, and to edit (modify after the fact). Go back to the second tangent calculation above (remember how to reposition the cursor!) and change it to compute the tangent of Pi / 3.

Do you remember the example in the first handout? (If not, it asked for the product of 123456 and 654321.) What Maple thinks the answer is. How does this compare with the result obtained by your calculator? Explain any differences.

```
>
```

Now let's define some functions, and see how Maple operates with these. (The only difference is that in these examples the name x does not have a value.)

```
>
```

```
> f := sin( 2  * x ) ;
```

$$f := \sin(2\,x)$$

```
> g := 4 * x^2 ;
```

$$g := 4\,x^2$$

```
> h := 0.25 * cos ( 8 * x ) ;
```

$$h := .25\,\cos(8\,x)$$

```
> p := f * g ;
```

$$p := 4\,\sin(2\,x)\,x^2$$

```
> u1 :=  f * h ;
```

$$u1 := .25\,\sin(2\,x)\,\cos(8\,x)$$

```
> u2 := f + h ;
```

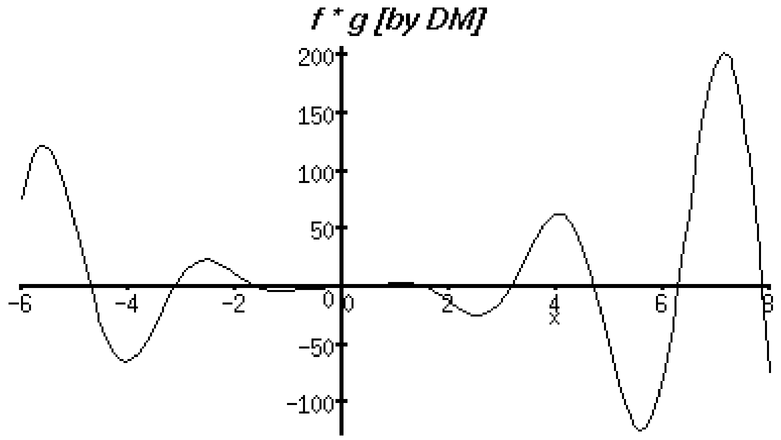
$$u2 := \sin(2\,x) + .25\,\cos(8\,x)$$

```
>
```

PLOTTING

Graphs of functions are produced by the plot command. In its simplest form, plot needs to know the function to be plotted and the range of values for the independent variable. Note that a..b is Maple's way of describing the interval [a,b]. Observe that I have placed signed titles on some of these graphs. When you actually print plots to turn in with your homework, they MUST have a title and signature [by your initials or last name]. Again, try to predict the output before tapping the return key!

```
>
> plot( 3 * t - 2 , t = -3 .. 10 ) ;
> plot( f , x = -Pi .. 2 * Pi ) ;
> plot( p , x = -6 .. 8 , title = ‘ f * g [by DM] ‘ );
```




```
>
```

These plots are nice, but what information do they convey? Let's concentrate on the plot of p, that is f*g. Position the cursor on a point on the graph and click the left button. The numbers that appear on the lower-left corner are the coordinates of the current location of the cursor. Use this technique to identify the global maximum and minimum values of f*g on the interval [-6, 8], and the x-values at which these are found. Where do other (local) minima and maxima occur? Can you guess the exact values?

```
>
> plot(2 *  u^3 + 4 * u - 5 , u = -10  .. 7 , title = ‘ cubic function [by DM] ‘ ) ;
>
```

Where does the cubic function cross the u-axis? Maybe it would be helpful to cut down the plotting interval from [-10, 7] to [-1, 3], or even narrower, say to [0, 1.5]. Try it. Can you find the u-intercept to 2-decimal point accuracy by this process? It is a powerful method, which we call ZOOMING IN.

```
>
> plot( u1 ,  x = -1.2 .. 1.2 );
> plot( u2 , x = 0 .. 3*Pi ) ;
>
```

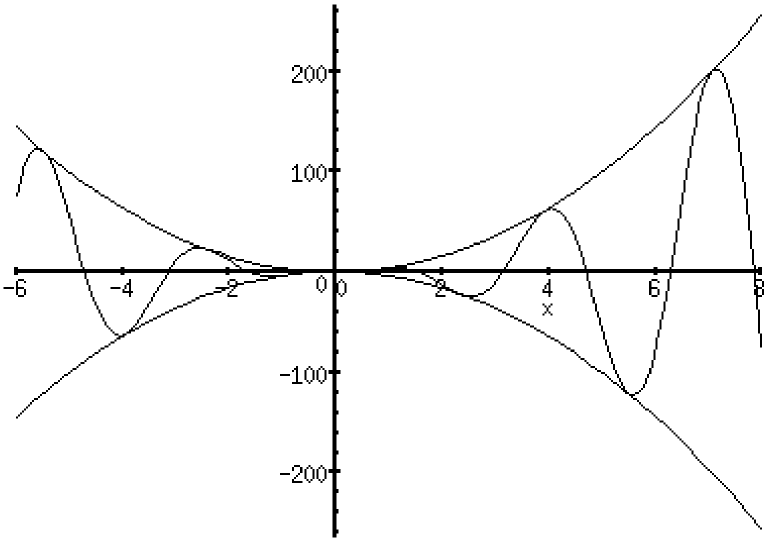
Can you explain why the graphs of u1 and u2 look the way they do?

You probably now have a big pile of plots cluttering up your screen. Ones that you don't need can be closed temporarily by minimizing the window. To permanently get rid of plots that you no longer need, position the cursor inside the plot window, select Exit from the File menu.

PLOTTING MULTIPLE FUNCTIONS IN ONE PLOT

To plot more than one function in a single window, just specify the functions to be plotted as a set.

> plot({ p , g , -g } , x= -6 .. 8);



>

Can you identify the three different plots? What about the next one? Can you explain the little bumps that appear inside the large dips in the graph of s?

> plot({ f , h , u2 }, x=0 .. 3*Pi);

>

A DISCONTINUOUS FUNCTION

The tangent function is well-known to all of us.

Note: This definition completely replaces the previous definition of f.

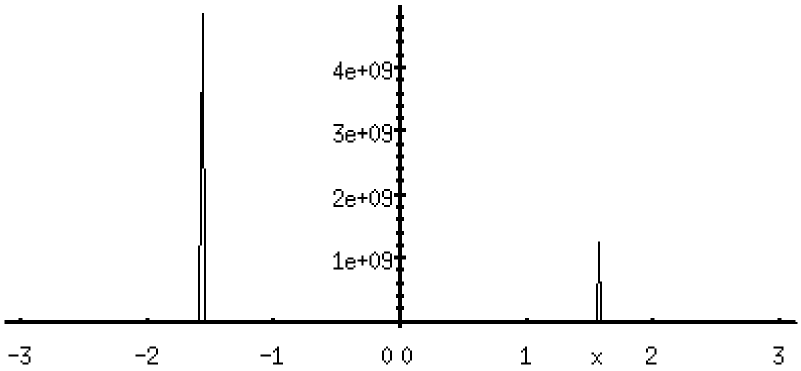
> f := tan(x);

$$f := \tan(x)$$

>

Recall that the graph of y=tan(x) has vertical asymptotes at all odd multiples of Pi/2. Let’s see how Maple handles this.

> plot(f , x = -Pi .. Pi);

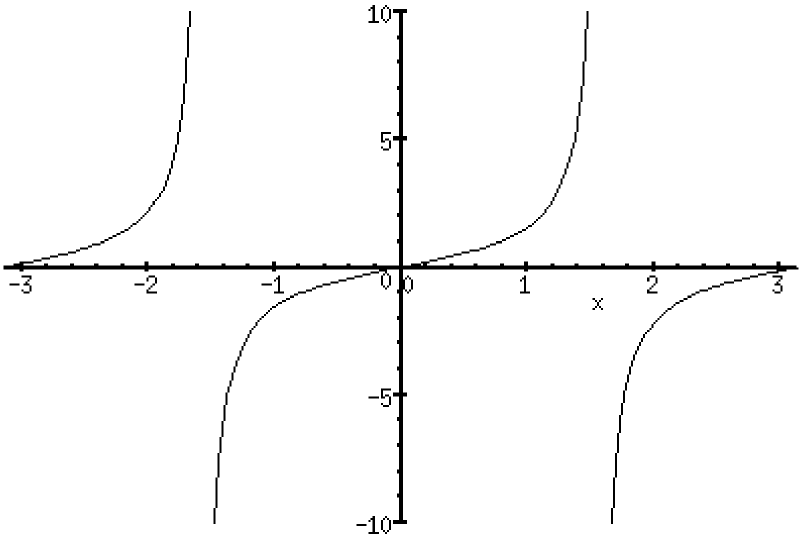


>

Is this what you expected to see? Why does the graph look like this? What can be done to improve the appearance of the plot?

There are two problems. First, we have to tell Maple that this function is discontinuous. But, there is more. Look at the labels on the vertical axis. The units are in BILLIONS; we want to see the details on a much finer scale. Let's limit the vertical scale to the range from -10 to 10. Each of these pieces of information is an optional argument to plot.

> plot(f, x = -Pi .. Pi , -10 .. 10 , discont=true);



>

This technique is very useful for any function that has vertical asymptotes.

OK. Now it's your turn.

Add appropriate optional arguments to the following command to produce a graph from which you can identify the local minimum and local maximum. (Don't forget the title.)

> plot ((10 - 4*t^3) / (1 - t^2) , t = -3 .. 5);

>

Now, one last challenge. Use your graph to estimate the asymptote for this function. Check your estimate by creating graphing the function and the asymptote in the same plot.

>

CONGRATULATIONS! You have just completed your first Maple worksheet.

We will return to this worksheet in Wednesday's class. So that you can keep a record of your work, you should save the revised file to disk. To do this, select File in the menu bar, and then either Save (which will destroy the original version of day1.ms) or Save As... (which will leave day1.ms untouched, and will request a new name for this modified version). Worksheet names should always end with .ms; also unless you really need the output it is usually a good idea to remove it before saving -- to do this, select Format, then Remove All, and finally Output.

>