> restart;
with(DEtools): with(plots):

Supplemental Maple Worksheet for Exam 2

Math 242, Section 1
Spring 1995

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WWW URL:
http://www.math.scarolina.edu/~meade/math242/worksheets/exam2.ms

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PROBLEM #4

Here are the velocities for the two stages of the jump.

> v1 := 981/40 * (1 - exp(-2/5*t));

$$v1 := \frac{981}{40} - \frac{981}{40} e^{(-2/5\,t)}$$

> v2 := 327/40 * (1 + 2*exp(24-6/5*t) - 3*exp(16-6/5*t));

$$v2 := \frac{327}{40} + \frac{327}{20} e^{(24 - 6/5\,t)} - \frac{981}{40} e^{(16 - 6/5\,t)}$$

a.
> vOPEN := subs(t=20, v1);

$$vOPEN := \frac{981}{40} - \frac{981}{40} e^{(-8)}$$

> evalf(vOPEN);

$$24.51677278$$

The velocity is approximately 24.517 m/sec when the chute opens.

b.

> dsolve({ diff(x(t), t) = v1, x(0)=0 }, x(t));

$$x(t) = \frac{981}{40}\,t + \frac{981}{16}\,e^{(-2/5\,t)} - \frac{981}{16}$$

> x1 := rhs(");

$$x1 := \frac{981}{40}\,t + \frac{981}{16}\,e^{(-2/5\,t)} - \frac{981}{16}$$

> xOPEN := subs(t=20, x1);

$$xOPEN := \frac{6867}{16} + \frac{981}{16}\,e^{(-8)}$$

> evalf(");

$$429.2080681$$

The parachutist has fallen approximately 429.208 m when the chute opens. Thus, there are still (approximately) 1579.792 m -- just under one mile -- until the return to terra firma.

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c.

> dsolve({ diff(x(t), t) = v2, x(20)=xOPEN }, x(t));

$$x(t) = \frac{327}{40}\,t - \frac{109}{8}\,e^{(24 - 6/5\,t)} + \frac{327}{16}\,e^{(16 - 6/5\,t)} + \frac{4469}{16} + \frac{327}{8}\,e^{(-8)}$$

> x2 := rhs(");

$$x2 := \frac{327}{40}\,t - \frac{109}{8}\,e^{(24 - 6/5\,t)} + \frac{327}{16}\,e^{(16 - 6/5\,t)} + \frac{4469}{16} + \frac{327}{8}\,e^{(-8)}$$

> tIMPACT := fsolve(x2=2000, t);

$$tIMPACT := 210.4799741$$

The total time for the jump is approximately 210.48 seconds, or 3:30.48 (minutes).

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d.

> vIMPACT := subs(t=tIMPACT, v2);

$$vIMPACT := \frac{327}{40} + \frac{327}{20}\,e^{(-228.5759689)} - \frac{981}{40}\,e^{(-236.5759689)}$$

> evalf(");

$$8.175000000$$

The impact occurred at a velocity of (approximately) 8.175 m/sec. (Would you like to land this fast?)

BONUS

Look at the velocity, v2. What happens to this expression as t increases?

> limit(v2, t=infinity);

$$\frac{327}{40}$$

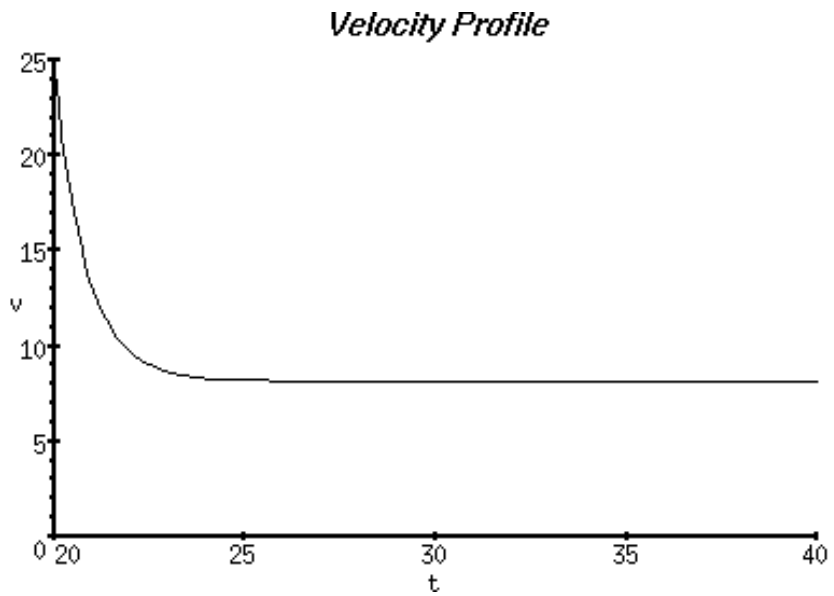
> evalf(");

$$8.175000000$$

The terminal velocity is larger than 8; that is, regardless of the initial velocity, ultimately the parachutist will descend at a rate of 8.175 m/sec. Since the velocity when the chute opens is larger than the terminal velocity, the velocity decreases to 8.175 m/sec, but no lower.

NO, the velocity never drops below 8 m/sec.

> plot(v2, t=20..40, v=0..25, title='Velocity Profile');



PROBLEM # 5: Improved Euler's Method

```
> f := (t,y) -> 1-sin(y) ;
  x[0] := 0 ;
  y[0] :=0 ;
  h := 0.015625 ;
  N := 256 ;
```

IMPROVED EULER'S METHOD for solving

$$y'(x) = f(x, y(x)), \quad y(x[0]) = y[0]$$

using N steps of size h.

```
> imp_euler_pts := [ x[0],  y[0] ]:
  for n from 0 to N-1 do
    F := f(x[n],y[n]);
    G := f(x[n]+h,y[n]+h*F);
    x[n+1] := x[n] + h;
    y[n+1] := y[n] + h/2*(F+G);
    new_pt := [ x[n+1], y[n+1] ];
    imp_euler_pts := imp_euler_pts, new_pt;
    print( new_pt );
  od:
>
```

h	N	y(4)	abs error	rel error
0.5	8	1.170057	---	---
0.25	16	1.174669	0.004612	0.003926
0.125	32	1.175690	0.001021	0.000868
0.0625	64	1.175929	0.000269	0.000229
0.03125	128	1.175986	0.000057	0.000049
0.015625	256	1.176000	0.000014	0.000012

Thus, the answers to the specific questions are:

a. The stepsize h=0.015625 is needed to achieve the requested accuracy.

b. The most accurate approximation is y(4) ~= 1.1760
(the first zero is correct; the other zeros are not known to be correct)

c. The relative error for the final approximation is 0.000012 = 1.2 E-5.
(Note that the errors are reduced by a factor of 4 with each refinement.)

END OF WORKSHEET

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>
```