>

MAPLE WORKSHEET for SEQUENCES (Section 11.1)

Math 142 -- Spring 1994

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This worksheet is intended to provide a graphical interpretation of sequences and their limits. The Sequence command provides the following information:

i) numerical values for the first N terms of the sequence;
ii) the limit of the sequence, if it converges;
iii) a plot of the first N terms of the sequence.

If you are interested, you can use Maple on the PCs in the public lab in PSC.

```
> with(plots):
```

First, we define a Maple procedure for providing exactly the information we desire.

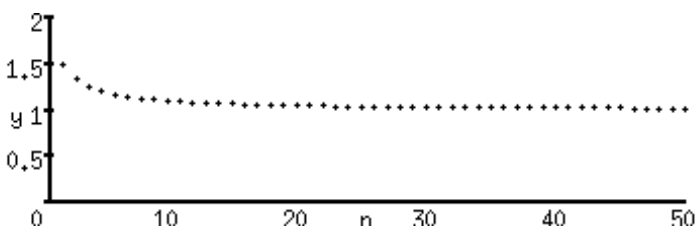
```
> Sequence := proc( expr, arg:name=range )
    local f, i, ii, S, var, lo, hi, m, M;
    var := lhs(arg); lo := op(1,rhs(arg) ); hi := op(2, rhs(arg) );
    f := unapply( expr, var );
    S := [seq( [i, evalf(f(i))], i=lo..hi )];
    m := min( 0, seq( op(2,s), s=S ) );
    M := max( 0, seq( op(2,s), s=S ) );
    print( S );
    print( Limit( f(var), var=infinity ) = limit( f(ii), ii=infinity) );
    plot( S, var=lo..hi, y=m..M, style=POINT );
  end:
Warning, 's' is implicitly declared local
```

Now for a few examples, the first is a simple problem to illustrate what is meant by the limit of the sequence.

```
> Sequence( 1+1/n, n=1..25 );
```

$$[[1, 2.], [2, 1.500000000], [3, 1.333333333], [4, 1.250000000],$$
$$[5, 1.200000000], [6, 1.166666667], [7, 1.142857143],$$
$$[8, 1.125000000], [9, 1.111111111], [10, 1.100000000],$$
$$[11, 1.090909091], [12, 1.083333333], [13, 1.076923077],$$
$$[14, 1.071428571], [15, 1.066666667], [16, 1.062500000],$$
$$[17, 1.058823529], [18, 1.055555556], [19, 1.052631579],$$
$$[20, 1.050000000], [21, 1.047619048], [22, 1.045454545],$$
$$[23, 1.043478261], [24, 1.041666667], [25, 1.040000000]]$$

$$\lim_{n \to \infty} \; 1 + \frac{1}{n} = 1$$



The next few examples illustrate the different behaviors that occur as the value of p changes in the sequences with

$$a_n := \frac{1}{n^p}$$

```
> Sequence( 1/n^1, n=1..50 );
> Sequence( 1/n^(1/2), n=1..50 );
> Sequence( 1/n^(1/10), n=1..50 );
> Sequence( 1/n^0, n=1..50 );
> Sequence( 1/n^(-1/10), n=1..50 );
> Sequence( 1/n^(-1/2), n=1..50 );
```

IMPORTANT FACT: These sequences converge if p>=0; they diverge if p<0.

Now, let's try a few examples that are a little more difficult.

```
> Sequence( (n^2+3*n)/(2*n^2+1), n=1..50 );
```

```
> Sequence( (3*n-1)/(n+sqrt(n)), n=1..50 );
```

```
> Sequence( ln(n)/exp(n), n=1..50 );
>
```

I hope the plots reinforce the difference between a sequence and a function.

Also, note that in some of the examples the limit is not really obvious by the 50th term. This is not uncommon. It is a reason why we should not become over-dependent on the computer (or calculator) and graphs. Sometimes the only way to really see what happens is to use the mathematical skills that we are in the process of developing.

```
>
```