MATH 141 (Section 5 & 6)
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University of South Carolina
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Quiz 8
October 24, 2013

Name: Key
Section: 005 / 006 (circle one)

1. (6 points) Find the absolute extreme values of the function $f(x) = (x^2+2x)^5$ on the interval $[-2, 1]$. Note: $2^5 = 32$, $3^5 = 243$, $4^5 = 1024$ and $5^5 = 3125$.

$f'(x) = 5(x^2+2x)^4(2x+2) = 10(x^2+2x)^4(x+1) = 0 \iff x^2+2x=0$ or $x+1=0$

$\iff x(x+2)=0$ or $x+1=0$

$\iff x=0,\ x=-2,$ or $x=-1$.

x	$f(x)$
0	0
-1	$(-1)^5=-1$
-2	0
1	$3^5=243$

abs. max : 243 @ $x=1$

abs. min : -1 @ $x=-1$.

2. (4 points) Verify that the function $f(x) = \frac{x}{x+4}$ satisfies the hypotheses of the Mean Value Theorem on $[-1, 2]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem. Note: $\sqrt{18} = 3\sqrt{2} \approx 4.2$.

Hypotheses of MVT: f(x) is continuous on $[-1,2]$

f(x) is differentiable on $(-1,2)$

$f'(x) = \frac{(x+4)(1) - 1 \cdot x}{(x+4)^2} = \frac{4}{(x+4)^2}$

$m_{sec} = \frac{f(2)-f(-1)}{2-(-1)} = \frac{\frac{2}{6} - \left(\frac{-1}{3}\right)}{2-(-1)} = \frac{1}{3}.$

solve: $f'(c) = \frac{1}{3}$

$\frac{4}{(x+4)^2} = \frac{1}{3}$

$(x+4)^2 = 4 \cdot 3 = 12$

$x+4 = \pm\sqrt{12} = \pm 2\sqrt{3}.$

$x = -4 \pm 2\sqrt{3} \approx -4 \pm 3.4 = \begin{cases} -0.6 \\ -7.4 \end{cases}$

$\therefore\ c = -4 + 2\sqrt{3} \approx -0.6$

Extra Credit (2 points) Give an example of a closed bounded interval $[a, b]$ on which the function $f(x) = \frac{x}{x+4}$ does not satisfy the conclusion of the Mean Value Theorem. Explain what hypothesis is not satisfied on your interval.

Any interval that includes $x = -4$.