MATH 141 (Section 3 & 4)
Prof. Meade

Quiz 1
August 29, 2013

University of South Carolina
Fall 2009

Name: __Key__

Section: (005) / 006 (circle one)

1. (4 points) Let $f(x) = \dfrac{1}{x}$. Evaluate the difference quotient $\dfrac{f(x) - f(a)}{x - a}$.

$$\frac{f(x) - f(a)}{x - a} = \frac{\frac{1}{x} - \frac{1}{a}}{x - a} = \frac{\frac{a}{ax} - \frac{x}{ax}}{x - a} = \frac{\frac{a - x}{ax}}{x - a} = \frac{a - x}{ax} \cdot \frac{1}{x - a} = \frac{-1}{ax}$$

2. (6 points) Let $f(x) = x + \dfrac{1}{x}$ and $g(x) = \dfrac{x + 2}{x + 1}$.

(a) Find $f \circ g$ and its domain.

$$(f \circ g)(x) = f(g(x)) = \frac{x + 2}{x + 1} + \frac{1}{\frac{x + 2}{x + 1}} = \frac{x + 2}{x + 1} + \frac{x + 1}{x + 2} = \frac{(x + 2)^2 + (x + 1)^2}{(x + 1)(x + 2)}$$

$$= \frac{x^2 + 4x + 4 + x^2 + 2x + 1}{(x + 1)(x + 2)} = \frac{2x^2 + 6x + 5}{(x + 1)(x + 2)}$$

$$\left(\text{domain: } x \neq -1 \quad x \neq -2 \right)$$

(b) Find $g \circ f$ and its domain.

$$(g \circ f)(x) = g(f(x)) = \frac{x + \frac{1}{x} + 2}{x + \frac{1}{x} + 1} = \frac{\frac{x^2 + 1 + 2x}{x}}{\frac{x^2 + 1 + x}{x}} = \frac{x^2 + 1 + 2x}{x} \cdot \frac{x}{x^2 + 1 + x} = \frac{x^2 + 2x + 1}{x^2 + x + 1}$$

$$\begin{cases} x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1}}{2 \cdot 1} = \frac{-1 \pm \sqrt{3} i}{2} \\ \text{So no real roots} \end{cases} \left(\text{domain: } (-\infty, \infty) \right)$$

Extra Credit (3 points) A car's trip odometer is reset. After driving d miles in t hours the onboard computer shows that the average speed is 14 mph and the fuel mileage is 19.1 miles per gallon. After driving another 385 miles in 6.5 hours the average speed is 35 miles per hour and the fuel mileage is 23.5 miles per gallon. How far has the car been driven since the trip odometer was reset?

HINT: $\dfrac{157.5}{21} = 7.5$.

$$\frac{d}{t} = 14 \implies d = 14t$$

$$\frac{d + 385}{t + 6.5} = 35 \quad \text{so} \quad \frac{14t + 385}{t + 6.5} = 35$$

$$14t + 385 = 35(t + 6.5)$$
$$= 35t + 227.5$$

$$21t = 385 - 227.5 = 157.5$$
$$t = \frac{157.5}{21} = 7.5 \text{ hr.}$$

$$d = 14t = 14(7.5) = 105 \text{ mi}$$